G. Toraldo di Francia (Ed.)

Onde superficiali

CIME Summer Schools

25

Varenna, Italy 1961

Springer

FONDAZIONE CIME ROBERTO CONTI

G. Toraldo di Francia (Ed.)

Onde superficiali

Lectures given at the
Centro Internazionale Matematico Estivo (C.I.M.E.),
held in Varenna (Como), Italy,
September 4-13, 1961

Springer

FONDAZIONE
CIME
ROBERTO CONTI

C.I.M.E. Foundation
c/o Dipartimento di Matematica "U. Dini"
Viale Morgagni n. 67/a
50134 Firenze
Italy
cime@math.unifi.it

ISBN 978-3-642-10981-2 e-ISBN: 978-3-642-10983-6
DOI:10.1007/978-3-642-10983-6
Springer Heidelberg Dordrecht London New York

Reprint of the 1st ed. C.I.M.E., Ed. Cremonese, Roma, 1961
With kind permission of C.I.M.E.

Printed on acid-free paper

Springer.com

CENTRO INTERNATIONALE MATEMATICO ESTIVO
(C.I.M.E)

Reprint of the 1st ed.- Varenna, Italy, September 4-13, 1961

ONDE SUPERFICIALI

PREMESSA

Le onde superficiali elettromagnetiche, pur essendo note da lungo tempo, hanno acquistato negli ultimi anni un'importanza notevole in un gran numero di applicazioni.

La loro teoria presenta problemi matematici di alto interesse. Non di rado si presentano anche curiose e dibattutissime difficoltà riguardo all'interpretazione fisica dei risultati matematici.

L'abbondante fioritura di studi sulle onde superficiali che si è avuta recentemente, si trova purtroppo sparsa nei periodici più disparati e riflette punti di vista molto diversi. Era sentitissimo il bisogno di una introduzione e di una messa a punto d'insieme per coloro che si vogliono dedicare all'argomento.

A questo scopo ha voluto rispondere il corso organizzato a Varenna dal Centro Internazionale Matematico Estivo dal 3 al 12 settembre 1961. In questi appunti, compilati dagli autori, sono condensate le lezioni del corso, che ebbe grande successo e fu accompagnato da molte interessanti discussioni.

Come coordinatore del corso tengo a ringraziare tutti gl'insegnanti che hanno portato il loro contributo e si sono sobbarcati alla fatica di mettere per iscritto le loro lezioni. Voglio anche rivolgere a nome di tutti gli studiosi della materia un vivo ringraziamento al C.I.M.E. ed in particolare al Direttore Prof. E.Bompiani ed al Segretario Prof. R.Conti per aver resa possibile la realizzazione del corso in modo così felice e proficuo.

Sono sicuro che queste lezioni rappresenteranno un contributo utilissimo alla letteratura internazionale su questo ramo della matematica applicata.

G. Toraldo di Francia

CENTRO INTERNAZIONALE MATEMATICO ESTIVO

(C.I.M.E.)

C. M. ANGULO

A DISCONTINUITY PROBLEM ON SURFACE WAVES : THE EXCITATION OF A GROUNDED DIELECTRIC SLAB BY A WAVEGUIDE .

ROMA - Istituto Matematico dell'Università

A DISCONTINUITY PROBLEM ON SURFACE WAVES :

The excitation of a grounded dielectric slab by a waveguide .

C. M. ANGULO

Institute for Defense Analyses, Washington D.C. +)

Introduction

The present discussion illustrates the solution of one discontinuity problem associated with the excitation of surface waves. The concepts developed in previous lectures by Zucker and Felsen are used repetedly throughout the discussion. One important point to emphasize is the usefullness of the modal analysis method which enables us to set up immediately the transform equation to apply the Wiener-Hopf technique.

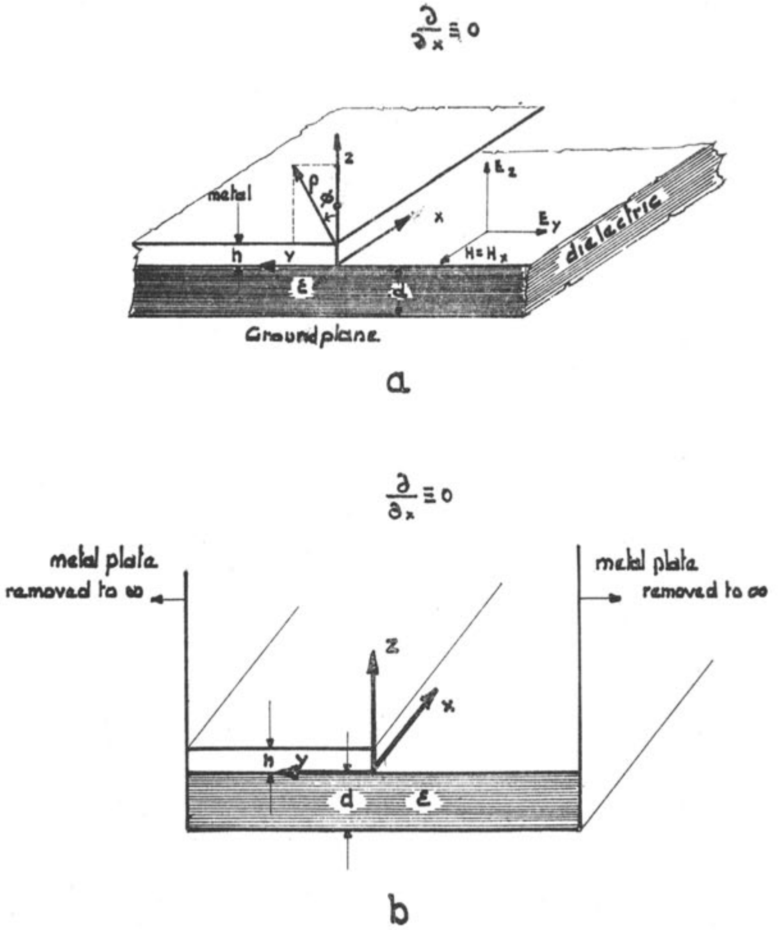
The problem is illustrated in figure 1 . The input energy is contained in the dominant TM ($H_y = 0$) mode of the partially filled waveguide propagating from $y = +\infty$ to $y = 0$. The dimensions of the guide and the thickness of the slab are restricted to the range for which only one surface wave (the lowest)[1] exists along the slab and only one mode (the dominant TM) can propagate inside the partially filled waveguide. These conditions are [2] :

$$Kd < \pi(\varepsilon)^{-1/2} \qquad (1a)$$

$$Kh < \arctan\left\{-(\varepsilon)^{-1/2}\tan\left[(\varepsilon)^{1/2}Kd\right]\right\} \qquad (1b)$$

$$0 < \arctan\left\{-(\varepsilon)^{-1/2}\tan\left[(\varepsilon)^{1/2}Kd\right]\right\} < \pi \qquad (1c)$$

+) On leave of absence from Brown University, Providence R.I.



∂/∂x ≡ 0
metal
z
ρ
φ
x
h
y
ε
d
Ez
Ey
H=Hx
dielectric
Groundplane
a
∂/∂x ≡ 0
metal plate
removed to ∞
metal plate
removed to ∞
z
x
h
y
d
ε
b

Fig. 1

where $K = \omega(\mu_0 \varepsilon_0)^{1/2}$, ε is the relative permittivity, d is the thickness of the dielectric, and d + h is the height of the parallel plate waveguide as indicated in Fig.1(a).

Because of the discontinuity at $y = 0$ where the upper plate is terminated, the energy incident upon the discontinuity will be partly reflected back into the waveguide, partly transmitted to the surface wave in the grounded dielectric slab and partly radiated. We are interested in finding the three power ratios for different values of Kd and Kh.

Since the structure shown in Fig.1(a) does not vary in the x direction and the incident wave is the lowest TM mode in the partially filled waveguide, all the fields excited will be independent of x and will have $E_x = H_y = H_z = 0$.

The structure shown in Fig.1(a) is regarded mathematically as a homogeneous parallel plate air waveguide (with walls at $y = \pm \infty$) and extending from $z = 0$ to $z = +\infty$ connected to a homogeneous parallel plate waveguide of length d, filled with dielectric of relative permittivity ε (also with walls at $y = \pm \infty$) and terminated by an electric wall at $z = -d$. Inside the first waveguide there is an obstacle, a semi-infinite perfectly conducting plane, placed at $z = h$ from $y = 0$ to $y = +\infty$. Fig.1(b) illustrates the above description. By removing the y = const. walls to infinity, the structure of Fig.1(a) is obtained.

The modal analysis of a parallel plate waveguide with walls at infinity represents the transversal fields (E_y and H_x, in our case) in terms of their Fourier transforms in the cross-section (variable y, in our case). In the successive sections we shall proceed as follows :

1) The equation relating the Fourier transforms of the fields at the semi-infinite obstacle will be derived by the modal analysis method.

2) The Wiener-Hopf technique will be applied to the solution of the equation obtained, and the exact fields will be expressed as the results of integrations in the complex plane.[3-5]

3) These integrals will be evaluated only at points far away from the discontinuity. The evaluation will be carried out by analyzing the relationship between the singularities of the integrands and the analytical forms of the far fields, which are known. In fact, for $y \ll 0$ the principal contribution on the surface of the slab must be the principal surface wave propagating along a grounded dielectric slab, and for $y \gg 0$ and $z < h$ the fields must be those of the dominant mode in a parallel plate waveguide partially filled with dielectric.

THE EQUATION FOR THE FOURIER TRANSFORMS OF THE FIELDS AT PLANE z = h

We first separate H_x and E_y into the incident and scattered fields; in a second step, we find the expressions for the Fourier transforms of the scattered fields for $z < h$ and $z > h$; and finally we match the boundary conditions at $z = h$ in terms of the Fourier transforms.

Let us represent the fields everywhere as :

$$H_x = H_{ox} + \mathcal{H}_x \tag{2a}$$

$$E_y = E_{oy} + \mathcal{E}_y \tag{2b}$$

where E_{oy} and H_{ox} are the components of the dominant TM mode of the partially filled parallel waveguide and $\mathcal{H}_x$ and $\mathcal{E}_y$ are the scattered fields. For $-d < z < h$ we have

$$H_{ox} = \left\{ \frac{\cosh\left[K(z-h)s'\right]}{\cosh(Khs')} u(z) + \frac{\cos\left[K(z+d)r'\right]}{\cos(Kdr')} u(-z) \right\} \times \exp\left[jK(1+s'^2)^{1/2} y\right] \tag{3a}$$

$$E_{oy} = -j \frac{Ks' \tanh(Khs')}{\omega \varepsilon \varepsilon_0} \left\{ \frac{\sinh\left[K(z-h)s'\right]}{\sinh(Khs')} u(z) - \frac{\sin\left[K(z+d)r'\right]}{\sin(Kdr')} u(-z) \right\} \exp\left[jK(1+s'^2)^{1/2} y\right] \tag{3b}$$

For $h < z$ we have

$$H_{ox} = 0 . \tag{3c}$$

$$E_{oy} = 0 . \tag{3d}$$

The time dependence is taken as $e^{i\omega t}$.

The function u(z) represents Heaviside's unit step function, zero for negative argument and one for positive arguments. The quantities r' and s' are the modulus of the wave numbers in the OZ direction in the dielectric and in the air respectively normalized with respect to K for the incident mode. They are the solutions of the following equations

$$r'^2 + s'^2 = \varepsilon - 1 \tag{4a}$$

$$\varepsilon \frac{s'}{r'} = \frac{\tan (Kdr')}{\tanh (Khs')} \tag{4b}$$

We will find below the quantity s similar to s'.s represents the modulus of the wavenumber in the OZ direction, in the air, normalized with respect to K for the lowest TM surface wave in a grounded dielectric slab.

The scattered fields can be represented by their Fourier transforms :

$$\mathcal{H}_x(y,z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} I(\eta,z)e^{-j\eta y} d\eta \tag{5a}$$

$$\mathcal{E}_y(y,z) = \frac{-1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} V(\eta,z)e^{-j\eta y} d\eta \tag{5b}$$

$$I(\eta,z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \mathcal{H}_x(y,z)e^{j\eta y} dy \tag{5c}$$

$$V(\eta,z) = \frac{-1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \mathcal{E}_y(y,z)e^{j\eta y} dy \tag{5d}$$

Maxwell's equations require that the transforms be solutions of the transmission line equations :

$$\frac{dV(\eta,z)}{dz} = -j\,\zeta\, ZI(\eta,z) \qquad (6a)$$

$$\frac{dI(\eta,z)}{dz} = -j\,\zeta\, YV(\eta,z) \qquad (6b)$$

for $z > h$ and for $h > z > -d$.

In the air,

$$\zeta = \zeta_a = (K^2 - \eta^2)^{1/2} \qquad (6c)$$

$$Y_a = \frac{1}{Z_a} = \omega\varepsilon_o(K^2 - \eta^2)^{-1/2} \qquad (6d)$$

In the dielectric

$$\zeta = \zeta_d = (K^2\varepsilon - \eta^2)^{1/2} \qquad (6e)$$

$$Y_d = \frac{1}{Z_d} = \omega\varepsilon_o\varepsilon(K^2\varepsilon - \eta^2)^{-1/2} . \qquad (6f)$$

If we recall Fig.1, we see that the solutions for V and I can be written immediately from the theory of transmission lines for the two regions $z > h$ and $z < h$, as follows : For $z > h$,

$$V(\eta,z) = V(\eta, h_+)\exp\left\{-j\,\zeta_a(z-h)\right\} \qquad (7a)$$

$$I(\eta,z) = Y_a V(\eta,h_+)\exp\left\{-j\,\zeta_a(z-h)\right\} . \qquad (7b)$$

For $h > z > 0$,

$$V(\eta,z) = V(\eta,h_-)\cos\zeta_a(z-h) -$$
$$-jZ_a I(\eta,h_-)\sin\zeta_a(z-h) \qquad (8a)$$

$$I(\eta,z) = I(\eta,h_-)\cos\zeta_a(z-h) -$$
$$-jY_a V(\eta,h_-)\sin\zeta_a(z-h) \qquad (8b)$$

Finally, for $0 > z > -d$,

$$V(\eta,z) = \left[V(\eta,h_-)\cos\zeta_a h + jZ_a I(\eta,h_-)\sin\zeta_a h\right]\cdot$$
$$\cdot\ \frac{\sin\zeta_d(z+d)}{\sin\zeta_d d} \qquad (8c)$$

$$I(\eta,z) = \left[I(\eta,h_-)\cos\zeta_a h + jY_a V(\eta,h_-)\sin\zeta_a h\right]\cdot$$
$$\cdot\ \frac{\cos\zeta_d(z+d)}{\cos\zeta_d d} \qquad (8d)$$

where

$$I(\eta,h_-) = -jY_a V(\eta,h_-)\ \frac{\frac{\zeta_d}{\varepsilon}\tan(\zeta_a h)\tan(\zeta_d d) - \zeta_a}{\frac{\zeta_d}{\varepsilon}\tan(\zeta_d d) + \zeta_a\tan(\zeta_a h)}. \qquad (8e)$$

The relationship between the values of V and I at $z = h_-$ and at $z = h_+$ are obtained from the boundary conditions of $\mathcal{H}_x$ and $\mathcal{E}_y$ at $z = h$. Let us first define the following new quantities :

$$V^+(\eta,h) = \frac{-1}{(2\pi)^{1/2}}\int_0^\infty \mathcal{E}_y(y,h)e^{j\eta y}dy \qquad (9a)$$

$$V^-(\eta,h) = \frac{-1}{(2\pi)^{1/2}}\int_{-\infty}^0 \mathcal{E}_y(y,h)e^{j\eta y}dy \qquad (9b)$$

$$\mathcal{J}^{+}(\eta,h) = \frac{1}{(2\pi)^{1/2}} \int_{0}^{\infty} \left[\mathcal{H}_x(y,h_+) - \mathcal{H}_x(y,h_-)\right] e^{j\eta y} dy \qquad (9c)$$

$$\mathcal{J}^{-}(\eta,h) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{0} \left[\mathcal{H}_x(y,h_+) - \mathcal{H}_x(y,h_-)\right] e^{j\eta y} dy \qquad (9d)$$

It is obvious that

$$V(\eta,h_+) = V(\eta,h_-) = V^{+}(\eta,h) + V^{-}(\eta,h) \qquad (10a)$$

and

$$I(\eta,h_+) - I(\eta,h_-) = \mathcal{J}^{+}(\eta,h) + \mathcal{J}^{-}(\eta,h). \qquad (10b)$$

Two constants will appear very often in the equations below, so for convenience we will represent them as follows :

$$a_1 = (1 + s^2)^{1/2} \qquad a_1 > 0$$

$$a_2 = (1 + s'^2)^{1/2} \qquad a_2 > 0 \ .$$

From the remaining boundary conditions at $z = h$, we obtain the following results :

$$\mathcal{E}_y = 0 \qquad \text{for} \qquad y > 0 \ , \qquad (11a)$$

therefore

$$V^{+}(\eta,h) = 0 \ ; \qquad (11b)$$

and

$$\mathcal{H}_x(y,h_+) - \mathcal{H}_x(y,h_-) = H_{ox}(y,h_-) \quad \text{for} \quad y < 0 \ , \qquad (12a)$$

therefore

$$\mathcal{J}^{-}(\eta,h) = -j \frac{\operatorname{sech}(Khs')}{(2\pi)^{1/2}\left[\eta + Ka_2\right]} \qquad (12b)$$

provided Imag η < Imag $(-Ka_2)$.

Therefore, all the boundary conditions at z = h are satisfied if

$$\mathcal{J}^+(\eta,h) = \frac{2\,\omega\varepsilon_0(K^2a_1^2 - \eta^2)}{(K^2-\eta^2)^{1/2}\,(K^2a_2^2-\eta^2)}\,G(\eta)V^-(\eta,h) + j\,\frac{\operatorname{sech}(Khs')}{(2\pi)^{1/2}\,(\eta + Ka_2)} \tag{13a}$$

where

$$G(\eta) = \frac{-j + \tan\left[h(K^2-\eta^2)^{1/2}\right]}{2} \times \frac{K^2a_2^2-\eta^2}{K^2a_1^2-\eta^2} \times \tag{13b}$$

$$\times \frac{1 + j\,\dfrac{(K^2\varepsilon-\eta^2)^{1/2}}{\varepsilon(K^2-\eta^2)^{1/2}}\tan\left[(K^2\varepsilon-\eta^2)^{1/2}d\right]}{\tan\left[(K^2-\eta^2)^{1/2}h\right] + \dfrac{(K^2\varepsilon-\eta^2)^{1/2}}{\varepsilon(K^2-\eta^2)^{1/2}}\tan\left[(K^2\varepsilon-\eta^2)^{1/2}d\right]}$$

The quantity s is the modulus of the wavenumber in the OZ direction in the air normalized with respect to K for the lowest TM surface wave in the grounded dielectric slab.

A study of the behavior of the functions in (13a) permits us to apply the Wiener-Hopf technique and solve for $\mathcal{J}^+$ and V^-.

THE SOLUTION OF THE EQUATION FOR THE TRANSFORMS

The behavior of $\mathcal{J}^{+}(\eta,h)$ and $V^{-}(\eta,h)$ in the complex η plane is determined by the asymptotic behavior of $\mathcal{H}_x$ and $\mathcal{E}_y$ as well as by the singularities of the transformed Kernel

$$\frac{2\,\omega\,\varepsilon_0(K^2 a_1^2 - \eta^2)}{(K^2 - \eta^2)^{1/2}(K^2 a_2^2 - \eta^2)}\,G(\eta)\ . \qquad (14)$$

We will come back later to (14). Let us proceed now with a physical derivation of the dominant terms of the far fields.

In our problem we can obtain all of the excited fields from the x component of the magnetic field. The problem may be compared with the two-dimensional field excited along a grounded dielectric slab by a magnetic line source along the OX axis. If $-\pi/2 < \phi_0 < 0$ and ρ is very large, we will not be able to notice any difference between a magnetic line and a terminated parallel plate waveguide propagating the lowest E mode. Therefore, the nature of the solution for both problems is the same for that region of space. However, as ϕ_0 increases, the angular dependence will be different for the two problems.

In the case of the magnetic line, we would have only the surface wave for $\phi_0 \approx -\pi/2$, all other terms being of order $\rho^{-3/2}$ or lower. As ϕ_0 increases, the surface-wave contribution becomes negligible and the dominant term varies like $\rho^{-1/2}$ (always for large ρ). Finally, when ϕ_0 grows to $\pi/2$, the $\rho^{-1/2}$

terms are not present and we have only terms of order $\rho^{-3/2}$ or lower and the surface wave.

For a grounded dielectric slab excited by a parallel plate waveguide, we will have also the surface wave and terms of order $\rho^{-3/2}$ and lower if $\phi_0 \approx -\pi/2$. As ϕ_0 increases, the surface wave contribution becomes negligible and the dominant term varies like $\rho^{-1/2}$. Finally, when ϕ_0 increases to $\pi/2$, the fields will go to zero as $\rho^{-3/2}$ at least and the surface wave will not reappear if we remain outside the waveguide.

It is therefore justified to write the form of the far fields for $K\rho \gg 1$ and $\pi/2 \geqslant \phi_0 \geqslant -\pi/2$ as follows :

$$H_x = \mathcal{H}_x = g(\phi_0)\frac{e^{-jK\rho}}{\rho^{1/2}}\cos\phi_0 + C\left[\frac{\cos\left[Kr(z+d)\right]}{\cos(Krd)}u(-z) + e^{-Ksz}u(z)\right]\Psi(\phi_0)\cdot\exp(jKya_1) \tag{15a}$$

$$E_y = \mathcal{E}_y = -\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} g(\phi_0)\frac{e^{-jK\rho}}{\rho^{1/2}}\cos\phi_0 + jsC\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}\left[\frac{\sin\left[Kr(z+d)\right]}{\sin(Krd)},u(-z) + e^{-Ksz}u(z)\right]\Psi(\phi_0)\exp(jKya_1) \tag{15b}$$

$$\Psi(\phi_0) = 0 \quad \text{if} \quad \frac{\phi_0}{2} > -\arctan(a_1 - s) \tag{15c}$$

$$\Psi(\phi_0) = 1 \quad \text{if} \quad \frac{\phi_0}{2} < -\arctan(a_1 - s) \tag{15d}$$

The transmission coefficient to the surface wave is represented by C. The new quantities ϕ_0 and ρ are the usual cylindrical coordinates illustrated in Fig.1(a) : r is the normalized wavenumber in the dielectric in the OZ direction for the lowest TM surface wave in the grounded dielectric slab. Finally $g(\phi_0)$ is a function of the observation angle ϕ_0 .

Inside the partially filled parallel plate waveguide and far away from the discontinuity, we have only the incident and reflected wave associated with the only propagating mode.

Therefore,

$$\mathcal{H}_x = BH_{ox} \exp\left\{-j2Kya_2\right\} \qquad (16a)$$

for $Ky \gg 1$ and $-d < z < h$;

$$\mathcal{E}_y = - BE_{oy} \exp\left\{-j2Kya_2\right\} \qquad (16b)$$

for $Ky \gg 1$ and $-d < z < h$.

B is the reflection coefficient. The quantities K , Ka_1 and Ka_2 must have small negative imaginary parts for dissipative media.

From a detailed examination of the singularities and zeros of (14) and from the asymptotic expression of the fields given in (15) and (16), it follows that $V^-(\eta,h)$ is analytic in the lower half of the η plane for Imag η < Imag $(-Ka_1)$ and its singularities are a branch point at $\eta = -K$ and a simple pole at $\eta = -Ka_1$.

It also follows that $\mathcal{J}^+(\eta,h)$ is analytic in the upper half of the η plane for Imag η > Imag (Ka_2) and its singularities are a branch point at $\eta = K$, a simple pole at $\eta = Ka_2$, and a countable infinite number of poles on the negative imaginary axis $\eta = -j|\eta_i|$ (*).

In a slightly dissipative medium, γ^+, V^- and $G(\eta)$ are analytic in a common narrow strip of width $2w_d$ along the real axis, where

$$0 < w_d < |\text{Imag } K| \tag{17a}$$

$$0 < w_d < |\text{Imag } (Ka_2)| \tag{17b}$$

$$0 < w_d < |\text{Imag } (Ka_1)| . \tag{17c}$$

The regions where γ^+ and V^- are analytic, their singularities relevant to the integration, the overlapping strip and the branch cuts, are shown in Fig.2. We restrict ourselves to remain on the Riemann sheet where

$$\text{Imag } (K^2 - \eta^2)^{1/2} < 0$$

We now decompose the function G:

$$G(\eta) = \exp\left\{ \gamma^-(\eta) - \gamma^+(\eta) \right\} \tag{18a}$$

where

$$\gamma^-(\eta) = \frac{-1}{2\pi j} \int_{-\infty + jw_d}^{\infty + jw_d} \frac{\ln G(\xi)}{\xi - \eta} \, d\xi \tag{18b}$$

(*) <u>Note</u>: $\pm j|\eta_i|$ are the roots of

$$\tan\left[h(K^2 - \eta^2)^{1/2}\right] + \frac{(K^2 \varepsilon - \eta^2)^{1/2}}{\varepsilon (K^2 - \eta^2)^{1/2}} \tan\left[(K^2 \varepsilon - \eta^2)^{1/2} d\right] = 0$$

encluding $\eta = \pm Ka_2$.

is analytic for

$$\text{Imag}\,\eta < w_d$$

and

$$\gamma^+(\eta) = \frac{-1}{2\pi j} \int_{-\infty - jw_d}^{\infty - jw_d} \frac{\ln G(\xi)}{\xi - \eta}\, d\xi$$

is analytic for Imag $\eta > -w_d$.

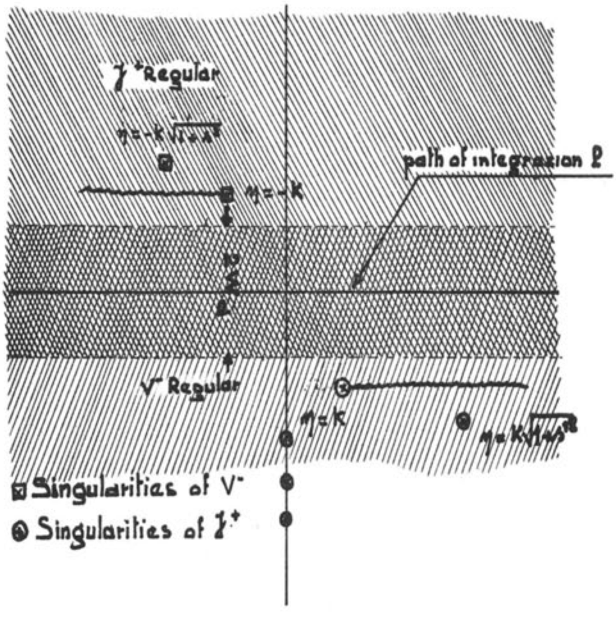


Fig.2 - The η-plane

Substituting (18a) into (13a) we can now group the terms with the same regions of regularity, as follows :

$$\mathcal{J}^{+}(\eta,h)\times\frac{(K-\eta)^{1/2}(\eta-Ka_2)\exp\left\{\gamma^{+}(\eta)\right\}}{\eta-Ka_1}$$

$$-j\exp\left\{\gamma^{+}(\eta)\right\}\frac{\operatorname{sech}(Khs')}{(2\pi)^{1/2}}\times\frac{(K-\eta)^{1/2}(\eta-Ka_2)}{(\eta+Ka_2)(\eta-Ka_1)}$$

$$+j\exp\left\{\gamma^{+}(-Ka_2)\right\}\frac{2\operatorname{sech}(Khs')}{(2\pi)^{1/2}}\times\frac{(K+Ka_2)^{1/2}}{\eta+Ka_2}\times\frac{a_2}{a_1+a_2}$$

$$=\frac{2\omega\varepsilon_0}{(K+\eta)^{1/2}}\times\frac{\eta+Ka_1}{\eta+Ka_2}\times V^{-}(\eta,h)\times\exp\left\{\gamma^{-}(\eta)\right\}$$

$$+j\exp\left\{\gamma^{+}(-Ka_2)\right\}\frac{2\operatorname{sech}(Khs')}{(2\pi)^{1/2}}\times\frac{(K+Ka_2)^{1/2}}{\eta+Ka_2}\times\frac{a_2}{a_1+a_2}$$

The last terms on each side of the previous equation are identical. They have been added in order to eliminate the pole at $\eta = -Ka_2$ of the left hand side of the equation.

The asymptotic behavior $\eta\to\infty$ of the unknown functions $\mathcal{J}^{+}$ and V^{-} in their respective regions of regularity is given by the behavior of the unknown field around the edge [6]. The physical requirement that the field scattered by the edge must have a finite amount of energy requires:

$$\frac{V^{-}(\eta,h)}{\sqrt{\eta}}\to 0 \quad\text{and}\quad \sqrt{\eta}\,\mathcal{J}^{+}(\eta,h)\to 0$$

as $\eta\to\infty$ in the region of regularity. These conditions are identical to those for the scattering of a plane wave by half a plane. [7]

We can now proceed with the customary reasoning of the Wiener-Hopf technique. The left hand side of the above equation is analytic in the upper half of the η plane including the narrow strip $j\left|\mathrm{Imag}\ \eta\right| < w_d$ and the right hand side is analytic in the lower half including the narrow strip. Therefore, they must be analytic continuation of each other representing an entire function of η. Furthermore, both sides approach zero as η approaches infinity. Thus, the entire function is zero. Equating both sides to zero we obtain

$$\mathcal{J}^{+}(\eta,h) = \frac{j\ \mathrm{sech}(Khs')}{(2\pi)^{1/2}(\eta + Ka_2)} \times \left\{1 - \frac{2a_2\left[K + Ka_2\right]^{1/2}(\eta - Ka_1)}{(K-\eta)^{1/2}(a_1 + a_2)(\eta - Ka_2)} \cdot \exp\left[\gamma^{+}(-Ka_2) - \gamma^{+}(\eta)\right]\right\} \quad (19a)$$

and

$$V^{-}(\eta,h) = \frac{-j\ \mathrm{sech}\ (Khs')}{\omega\varepsilon_0(2\pi)^{1/2}(\eta + Ka_1)} \times \quad (19b)$$

$$\times \frac{(K+\eta)^{1/2}\, a_2(K + Ka_2)^{1/2}}{a_2 + a_1} \exp\left\{\gamma^{+}(-Ka_2) - \gamma^{-}(\eta)\right\}$$

CALCULATION OF THE FAR FIELDS

The "voltage" is now known for $z = h$, since

$$V(\eta, h) = V^{-}(\eta, h) \quad . \tag{20a}$$

As for the "current" transform, we recall (7b) for $z = h$:

$$I(\eta, h_{+}) = Y_a V(\eta, h) \tag{20b}$$

Therefore $I(\eta, h_{+})$ is known. Moreover, (10b), (12b) and (19a) yield $I(\eta, h_{-})$, once $I(\eta, h_{+})$ has been found. Therefore, $I(\eta, h_{+})$ and $I(\eta, h_{-})$ are now both known.

The knowledge of V and I for $z = h$ gives us the expressions for V and I anywhere, as indicated in (7) and (8).

Finally, the inversion of V and I by (5) yields the exact expressions for the fields everywhere.

However, this solution everywhere is only formal, since the inversion of the transforms is practically impossible. Nevertheless, we can obtain all the information that we want by carrying out the inversion for the far fields at points of observation for which the method of steepest descents is easier to apply, and comparing the results with (15) and (16). In this way we can obtain the expressions for the coefficients B, C and $g(\phi_0)$ which give us the complete knowledge of the far fields.

The inverse transforms of special interest to us are :

$$E_y(y, h) = \mathcal{E}_y(y, h) = \frac{-1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} V^{-}(\eta, h) e^{-j\eta y} d\eta \tag{21a}$$

for z = h, y < 0 ;

$$H_x(y, h_+) - H_x(y, h_-) + H_{ox}(y, h) = \mathcal{H}_x(y, h_+) - \mathcal{H}_x(y, h_-) =$$

$$= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \left[\mathcal{J}^+(\eta, h) - \frac{j \operatorname{sech}(Khs')}{(2\pi)^{1/2}(\eta + Ka_2)} \right] e^{-j\eta y} d\eta \quad (21b)$$

for z = h, y > 0; and

$$E_y(y,z) = \mathcal{E}_y(y,z) = \frac{-1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} V^-(\eta, h) e^{-j\eta y} \exp\left\{-j(K^2-\eta^2)^{1/2}(z-h)\right\} d\eta \quad (21c)$$

for z > h.

The path of integration is indicated in Fig.2.

The integrals are evaluated for the far fields by the method of steepest descents for the limiting case of zero dissipation. For convenience these integrations are not carried out in the η plane but in a new ν plane. The coordinates are also changed from cartesian to polar coordinates, as illustrated in Fig.1(a).

$$\eta = K \sin\nu \qquad (K^2 - \eta^2)^{1/2} = K \cos\nu \quad (22a)$$

$$y = \rho \sin \phi_0 \qquad z - h = \rho \cos \phi_0 . \quad (22b)$$

With this change, the integrals (21) become

$$E_y(y, h) = \frac{-K}{(2\pi)^{1/2}} \int_\Lambda V^-(K \sin\nu, h) . \exp\left\{-jK\rho \cos(\nu + \frac{\pi}{2})\right\} \cos\nu \, d\nu \quad (23a)$$

for z = h, y < 0 ;

$$H_x(y, h_+) - H_x(y,h_-) + H_{ox}(y,h_-) =$$

$$= \frac{K}{(2\pi)^{1/2}} \int_{\Lambda} \left[\gamma^+(K \sin \nu, h) - j \frac{\operatorname{sech}(Khs')}{K(2\pi)^{1/2}(\sin \nu + a_2)} \right] \times$$

$$\times \cos \nu \quad \exp\left\{ - K \rho j \cos(\nu - \frac{\pi}{2}) \right\} d\nu \qquad (23b)$$

for $z = h$, $y > 0$; and

$$E_y(y, z) = \frac{-K}{(2\pi)^{1/2}} \int_{\Lambda} V^-(K \sin \nu, h) \times$$

$$\times \exp\left\{ -jK \rho \cos(\nu - \phi_0) \right\} \cos \nu \, d\nu \qquad (23c)$$

for $z > h$.

The path of integration is shown in Fig.3.

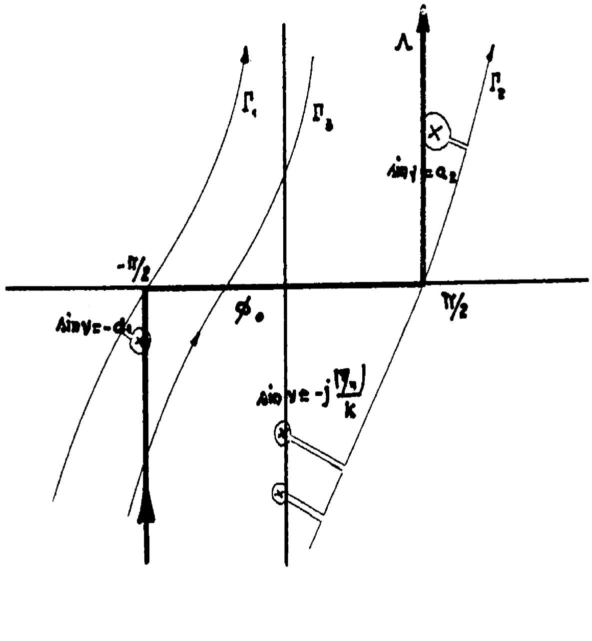


Fig.3 - The ν -plane

The saddle point for (23a) is obviously at $\nu = -\pi/2$, and the steepest descent path is the path Γ_1 in Fig.3. The integration indicated in (23a) is carried out along the path Γ_1 and not along the path Λ. The result is equal to $2\pi j$ times the residue of the integrand at the pole $\sin \nu = -a_1$ plus the asymptotic series obtained from the steepest descent integration. However, because of the factor $\cos \nu$, the $\rho^{-1/2}$ order contribution of the expansion is zero since $\cos(-\pi/2) = 0$. Therefore, if we neglect terms of order $\rho^{-3/2}$ or lower we can write for large ρ, (i.e., $y \ll 0$)

$$E_y(y,h) = -(2\pi)^{1/2} je^{jKa_1 y} \lim_{\eta \to -Ka_1} \left\{ (\eta + Ka_1) V^-(\eta,h) \right\}. \quad (24a)$$

This represents the surface wave excited along the grounded slab.

If we evaluate (15b) for the given observation point

$$z = h \qquad \phi_o = -\frac{\pi}{2}$$

$$y \ll 0 \qquad \rho \text{ very large,}$$

we obtain

$$E_y(y,h) = jsC \left(\frac{\mu_o}{\varepsilon_o} \right)^{1/2} e^{-Ksh} e^{jKa_1 y} \quad (24b)$$

Equating the right hand members (24), one obtains :

$$C = -\frac{(2\pi)^{1/2}}{s} \left(\frac{\varepsilon_o}{\mu_o} \right)^{1/2} \lim_{\eta \to -Ka_1} \left\{ (\eta + Ka_1) V^-(\eta,h) \right\} e^{Ksh}$$

The evaluation of the limit yields finally :

$$C = j\,\frac{a_2(1-a_1)^{1/2}(1+a_2)^{1/2}}{s(a_1+a_2)} \times \operatorname{sech}(Khs') \times$$

$$\times \exp\left\{ Khs + \gamma^{+}(-Ka_2) - \gamma^{-}(-Ka_1) \right\} . \qquad (25a)$$

The saddle point for (23b) is at $\nu = \pi/2$ and the steepest descent path is Γ_2 in Fig.3. The residues at the poles on the imaginary axis of the ν plane decay exponentially with an increasing positive y and are negligible for $Ky \gg 1$. These poles correspond to the non-propagating ordinary modes in the parallel plate waveguide with the dielectric slab, and we expected a negligible contribution for $Ky \gg 1$. The asymptotic expansion does not contain a term of order $\rho^{-1/2}$ for identical reasons that the integral discussed in the previous paragraph did not. The dominant contribution to the integral (23b) is therefore :

$$\mathcal{H}_x(y, h_+) - \mathcal{H}_x(y,h_-) = -(2\pi)^{1/2} j e^{-jKa_2 y} \times$$

$$\times \lim_{\eta \to Ka_2} \left\{ (\eta - Ka_2) \left[\mathcal{J}^{+}(\eta, h) - \frac{j\,\operatorname{sech}(Khs')}{(2\pi)^{1/2}(\eta + Ka_2)} \right] \right\} \qquad (26a)$$

where only terms of order $\rho^{-3/2}$ or lower have been neglected and $Ky \gg 1$. The expression (26a) clearly represents the reflected surface wave in the parallel plate waveguide evaluated at the upper plate.

If we evaluate (15a) and (16a) at the point of observation

$$z = h \qquad \phi_0 = \frac{\pi}{2}$$