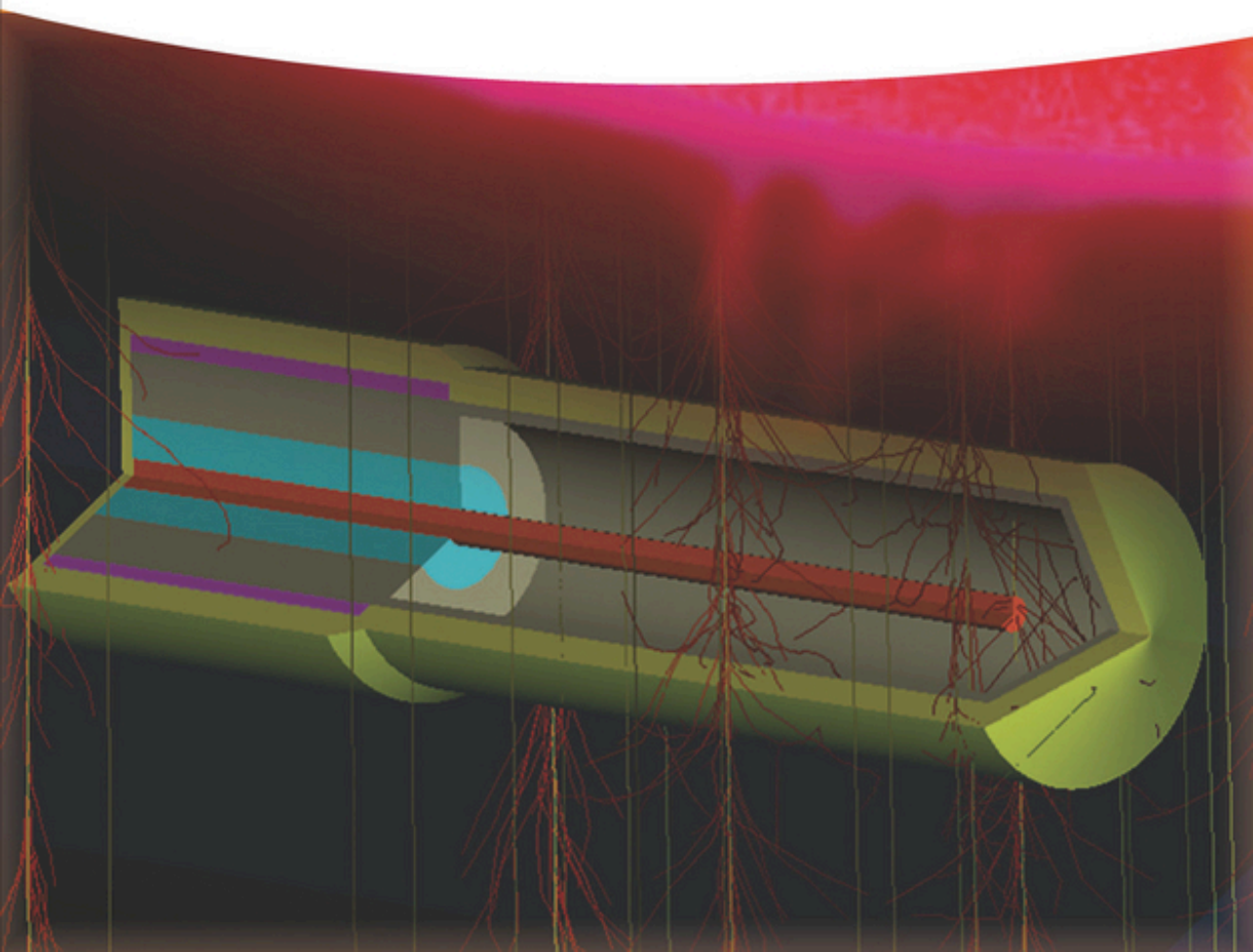
WILEY-VCH

Pedro Andreo, David T. Burns,
Alan E. Nahum, and Jan Seuntjens

Fundamentals of Ionizing Radiation Dosimetry

Solutions to the Exercises



Fundamentals of Ionizing Radiation Dosimetry

Solutions to Exercises

Fundamentals of Ionizing Radiation Dosimetry

Solutions to Exercises

Pedro Andreo, David T. Burns,
Alan E. Nahum, and Jan Seuntjens

WILEY-VCH

The Authors

Prof. Pedro Andreo, FInstP, CPhys
Karolinska University Hospital
171 76 Stockholm
Sweden

Dr. David T. Burns, FInstP
Bureau International des
Poids et Mésures
Pavillon de Breteuil
92312 Sévres Cedex
France

Prof. Alan E. Nahum, FIPEM
Visiting Professor
Physics Department
University of Liverpool
United Kingdom

Prof. Jan Seuntjens, FCCPM, FAAPM, FCOMP
McGill University
Medical Physics Unit
Cancer Research Program
Research Institute
McGill University Health Centre
1001 Décarie Blvd
Montreal QC H4A 3J1
Canada

Cover Credits
The cover image was kindly provided by Dr Jörg Wulff.

Library of Congress Card No.:
applied for

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at <http://dnb.d-nb.de.>

Print ISBN: 978-3-527-34352-2
ePDF ISBN: 978-3-527-81106-9
ePub ISBN: 978-3-527-81104-5
Mobi ISBN: 978-3-527-81105-2

Typesetting SPi Global, Chennai, India
Printing and Binding

Printed on acid-free paper

Contents

Preface *vii*

1 **Background and Essentials** *1*

2 **Charged Particle Interactions** *5*

3 **Uncharged Particle Interactions with Matter** *23*

4 **Field and Dosimetric Quantities and Radiation Equilibrium: Definitions and Interrelations** *35*

5 **Elementary Aspects of the Attenuation of Uncharged Particles through Matter** *47*

6 **Macroscopic Aspects of the Transport of Radiation through Matter** *53*
Implementation *54*
Normalization of Results *55*

7 **Characterization of Radiation Quality** *57*

8 **The Monte Carlo Simulation of the Transport of Radiation through Matter** *69*

9 **Cavity Theory** *85*

10 **Overview of Radiation Detectors and Measurements** *93*

11 **Primary Radiation Standards** *99*

12 **Ionization Chambers** *109*

13 **Chemical Dosimeters** *117*

14 **Solid-State Dosimeters** *123*

15 **Reference Dosimetry for External Beam Radiation Therapy** *129*

16 **Dosimetry of Small and Composite Radiotherapy Photon Beams** *143*

17 **Reference Dosimetry for Diagnostic and Interventional Radiology** *145*

18 **Absorbed-Dose Determination for Radionuclides** *153*

19 **Neutron Dosimetry** *169*

Preface

The first edition of Frank Herbert Attix's widely used book *Introduction to Radiological Physics and Radiation Dosimetry* was published in 1986 and reprinted in 2004. The exercises and solutions at the end of each chapter were widely regarded as a useful complement to the theory described in the different chapters.

In the second edition of the book, which we abbreviate as FIORD (from *Fundamentals of IOnizing Radiation Dosimetry*), the exercises have been updated and new ones prepared for chapters and topics that were not included in the first edition. Publishing the solutions to the exercises as a separate book was considered to be a more convenient approach, and they are presented here in order to complement some of the (sometimes limited) discussions in the textbook, supported by references to its equations and figures. Hopefully, they will also be a source of inspiration for teachers to prepare new exercises.

The electronic Data Tables of the textbook, necessary for the solution of the exercises, are available from http://www.wiley-vch.de/ISBN9783527409211; physical constants and atomic data are also given in Appendix A of the textbook.

6th December 2016

Pedro Andreo
David T. Burns
Alan E. Nahum
Jan Seuntjens

1

Background and Essentials

1 What is the photon energy range corresponding to the UV radiation band?
Answer: 10 nm–400 nm corresponds to 124 eV–3.1 eV.

Solution:
The quantum energy k of any electromagnetic photon is given in keV by

$$k = h\nu = \frac{hc}{\lambda} = \frac{12.3982 \text{ keV Å}}{\lambda} = \frac{1.23982 \text{ keV nm}}{\lambda}$$

where 1 Å(Angstrom) $= 10^{-10}$ m, Planck's constant is $h = 6.62607 \times 10^{-34}$ J s $= 4.13561 \times 10^{-18}$ keV s (note that 1.6022×10^{-16} J = 1 keV), and the velocity of light in vacuum is $c = 2.99792 \times 10^{8}$ m/s $= 2.99792 \times 10^{18}$ Å/s $= 2.99792 \times 10^{17}$ nm s^{-1}.
Therefore for the UV radiation, which is in the range of 10 nm–400 nm, the equation yields 124 eV–3.1 eV.

2 For a kinetic energy of 100 MeV, calculate the velocity β, for (a) electrons, (b) protons, and (c) alpha particles. The corresponding rest energies are given in the Data Tables.
Answer: (a) 0.9999; (b) 0.4282; (c) 0.2271

Solution:
We can apply either of the relations

$$\beta^2 = \frac{\tau(\tau+2)}{(\tau+1)^2}, \quad \text{with } \tau = E/m_0c^2$$

or

$$\beta^2 = \frac{E(E+2m_0c^2)}{(E+m_0c^2)^2}$$

From the Data Tables, the rest energies are $m_ec^2 = 0.51099$ MeV, $m_pc^2 = 938.272$ MeV, and $m_\alpha c^2 = 3727.38$ MeV. These yield
(a) Electrons: 0.9999
(b) Protons: 0.4282
(c) Alpha particles: 0.2271

Fundamentals of Ionizing Radiation Dosimetry: Solutions to Exercises, First Edition.
Pedro Andreo, David T. Burns, Alan E. Nahum, and Jan Seuntjens.

3 Conversely given a value of $\beta = 0.95$, calculate the corresponding kinetic energies of electrons, protons, and α particles.
Answer: (a) 1.1255 MeV; (b) 2066.6 MeV; (c) 8209.86 MeV

Solution:
The relation between the kinetic energy and the speed (β) is

$$E = \frac{m_0 c^2 \beta^2}{2\sqrt{1-\beta^2}}$$

Using the rest energies from the previous exercise, we get
(a) Electrons: 1.1255 MeV
(b) Protons: 2066.6 MeV
(c) α-particles: 8209.86 MeV

4 The result of a given process is derived as the product of several independent quantities, $Q = \prod q_i$. The type A and B uncertainties of each q_i, $(u_A, u_B)_i$, given as a relative standard uncertainty, are (0.1, 0.5), (0.01, 0.1), (0.02, 0.4), and (0.3, 0.19). Determine the combined standard uncertainty of Q.
Answer: $u_c(Q) = 0.75$

Solution:
Use the *law of propagation of uncertainty* twice: first for each of the respective types of uncertainty to yield the overall u_A and u_B types,

$$u_A = \sqrt{\sum_i u_{A_i}^2}, \quad u_B = \sqrt{\sum_i u_{B_i}^2}$$

and then for the combination of these two to yield $u_c(Q)$.
Hence

Quantity	Rel standard uncertainty	
	$(u_A)_i$	$(u_B)_i$
q_1	0.10	0.50
q_2	0.01	0.10
q_3	0.02	0.40
q_4	0.30	0.19
Combined	$u_A = 0.32$	$u_B = 0.68$

resulting in a combined uncertainty

$$u_c(Q) = \sqrt{u_A^2 + u_B^2} = 0.75$$

5 Given the following set of data (75.4, 79.7, 75.0, 77.0, 78.4), with standard uncertainties (0.95, 0.5, 0.2, 1.2, 0.8), (a) determine the non-weighted and weighted means and the corresponding type A uncertainties. (b) Determine the Birge ratio for the data and comment on the uncertainty estimates of the data.
Answer: $\bar{x} = 77.1$, $s_{\bar{x}} = 0.89$; $\bar{x}_w = 75.8$, $s_{\bar{x}_w} = 0.18$; $R_{Birge} = 2.2$

Solution:

(a) Requires the straightforward application of Eqs. (1.41)–(1.46), where the different terms are

i	x_i	$(x_i - \bar{x})^2$	s_i	$w_i(= 1/s_i^2)$	$w_i\, x_i$	$w_i(x_i - \bar{x}_w)^2$
1	75.4	2.89	0.95	1.11	83.55	0.18
2	79.7	6.76	0.50	4.00	318.80	60.79
3	75.0	4.41	0.20	25.00	1875.00	16.07
4	77.0	0.01	1.20	0.69	53.47	1.00
5	78.4	1.69	0.80	1.56	122.50	10.55
n	$\sum_i x_i$	$\sum_i (x_i - \bar{x})^2$		$\sum_i w_i$	$\sum_i w_i\, x_i$	$\sum_i w_i (x_i - \bar{x}_w)^2$
5	385.5	15.8		32.36	2453.32	88.58
Eqs	(1.41)	(1.43)		(1.45)	(1.44)	(1.46) num
	$\bar{x}$	$s(\bar{x})$		$s(\bar{x}_w)_{\text{int}}$	$\bar{x}_w$	$s(\bar{x}_w)_{\text{ext}}$
	77.10	0.89		0.18	75.80	0.83

(b) The Birge ratio is given by

$$R_{\text{Birge}} = \frac{s(\bar{x}_w)_{\text{int}}}{s(\bar{x}_w)_{\text{ext}}} = 2.2$$

$R_{\text{Birge}} = 2.2$ is a sign that some uncertainties have been under/over estimated. We typically think that we can make estimates at, say, the 20% level. A Birge significantly greater than 1.2 or 1.3 is a reasonable sign of under/overestimation. However, one proviso is the balance of uncertainties. One huge under/overestimate can make Birge large even if other uncertainties are properly estimated, especially for small data sets. This could be the case with data #3, where $s = 0.20$ might be an underestimation.

6 Using the half-width of the set of data in the previous exercise, estimate the type B uncertainty assuming rectangular, triangular, and Gaussian (with $k = 2$) distributions. Which of the three is considered to be more conservative?
Answer: $u_{B\,rect} = 1.36$, $u_{B\,trian} = 0.96$, $u_{B\,Gauss} = 1.18$; the 95% Gaussian is more conservative.

Solution:

The half-width of the set of data, $[-L, +L]$, is determined as

$$L = \frac{\max(x_i) - \min(x_i)}{2} = \frac{79.7 - 75.0}{2} = 2.35$$

Hence

$$u_{\text{B,rect}} = \frac{L}{\sqrt{3}} = \frac{2.35}{1.73} = 1.36$$

$$u_{\mathrm{B,trian}} = \frac{L}{\sqrt{6}} = \frac{2.35}{2.45} = 0.96$$

$$u_{\mathrm{B,95\%}} = \frac{L}{2} = \frac{2.35}{2} = 1.18$$

The rectangular distribution is a special case, because in general for most data sets there is a higher probability that the true value lies nearer to the middle than at the extremes. This leaves the triangular and Gaussian ($k = 2 \rightarrow 95\%$) distributions being conceptually similar, with the 95% Gaussian being more conservative.

2

Charged Particle Interactions

1 For the Rutherford (Geiger and Marsden) experiment with 5.5 MeV α particles on a 1 μm gold foil and for the six angles (decades) between 10^{-5} and 10^{0} rad, calculate the Rutherford differential cross section (DCS), $d\sigma_R/d\Omega$, (a) without and (b) with screening. Represent both results graphically and draw conclusions.
Answer: The non-screened DCS values vary between 1.7137×10^{-3} and 2.0273×10^{-23} cm^2 rad^{-1} in the interval [10^{-5}–1 rad]. The screening angle is 3.7×10^{-3} rad, and the corresponding screened DCS values vary between 9.3620×10^{-14} and 2.0273×10^{-23} cm^2 rad^{-1}. The screening χ_a cuts off the otherwise increasing DCS with decreasing angle, which remains practically constant below χ_a.

Solution:
Note that the foil thickness is irrelevant, as the interaction is assumed to be with one atom of gold, $Z = 79$. The α-particle charge is $z = 2$.

(a) The non-screened Rutherford DCS is given by

$$\frac{d\sigma_R}{d\Omega} = r_e^2(z\,Z)^2\left(\frac{m_e c^2}{m_0 c^2}\right)^2\frac{1-\beta^2}{\beta^4}\frac{1}{(1-\cos\,\theta)^2}$$

where the relevant constants given in the Data Tables are $m_e c^2 = 0.51099$ MeV, $m_0 c^2 = m_\alpha c^2 = 3727.38$ MeV, $r_e = 2.81794\times10^{-13}$ cm, and $\beta^2 = \tau(\tau+2)/(\tau+1)^2$ with $\tau = E/m_\alpha c^2$. This yields $\beta = 0.05426$. The DCS can then be calculated; the results are given in the table below.

(b) The screened Rutherford DCS is given by

$$\frac{d\sigma_R}{d\Omega} = (z\,Z\,r_e)^2\left(\frac{m_e c^2}{m_0 c^2}\right)^2\frac{1-\beta^2}{\beta^4}\frac{1}{(1-\cos\theta+0.5\chi_a^2)^2},$$

which differs from the non-screened DCS in the screening angle χ_a in the denominator. This is taken to be that of Molière, which is given by

$$\chi_a^2 = \chi_0^2\left[1.13 + 3.76\left(\frac{z\,Z}{137\beta}\right)^2\right]$$

Fundamentals of Ionizing Radiation Dosimetry: Solutions to Exercises, First Edition.
Pedro Andreo, David T. Burns, Alan E. Nahum, and Jan Seuntjens.

where

$$\chi_0 = \frac{4.2121 \times 10^{-3}\sqrt{1-\beta^2}}{m_0 c^2 \beta} Z^{1/3}$$

Thus the screening values obtained are

$$\chi_0 = 8.922 \times 10^{-5} \text{ rad} \quad (0.005^\circ)$$
$$\chi_a = 3.678 \times 10^{-3} \text{ rad} \quad (0.211^\circ),$$

which can be inserted in the screened DCS and its values obtained. Putting together the results for the two DCS, we get the following table:

	Rutherford DCS (cm^2 rad^{-1})	
θ(rad)	Non-screened	Screened
10^{-5}	1.7137×10^{-3}	9.3620×10^{-14}
10^{-4}	1.7137×10^{-7}	9.3483×10^{-14}
10^{-3}	1.7137×10^{-11}	8.1178×10^{-14}
10^{-2}	1.7137×10^{-15}	1.3296×10^{-15}
10^{-1}	1.7165×10^{-19}	1.7119×10^{-19}
10^{0}	2.0273×10^{-23}	2.0273×10^{-23}

which is shown in Figure 2.1 and shows the influence of screening, which at about 4×10^{-3} rad cuts the otherwise increasing DCS with angle, remaining practically constant below χ_a.

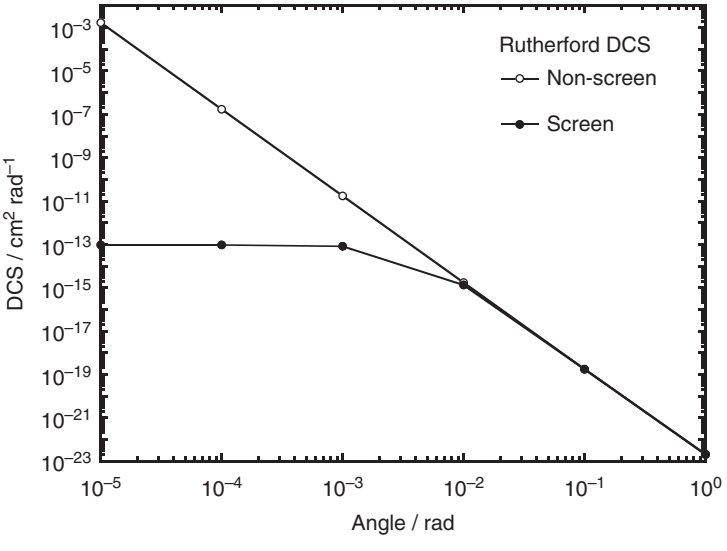


Figure 2.1 Rutherford differential cross section, with and without screening, for 5.5 MeV α-particles on a 1 μm gold foil.

2 Assume the Rutherford experiment to have been made with protons having the same speed (β) than the original 5.5 MeV α particles. Calculate the screened Rutherford DCS for the same angles as above. Compare graphically with the case of α particles from the previous exercise and draw conclusions.

Answer: In this case the screening angle is $\sim 7 \times 10^{-3}$ *rad, almost double than that for α particles with the same β, and the DCS varies between* 2.3635×10^{-14} *and* 7.9977×10^{-23} *in the interval* $[10^{-5}–1$ *rad]. The DCS of protons is, above the screening angle, larger than that of the α particles, as expected from the lower proton rest energy. The larger screening angle for protons, due also to the* $1/m_0c^2$ *dependence of* χ_0*, cuts off the DCS earlier (when decreasing angle) than for α particles, so that for* $\theta < \chi_a$*,* $DCS_{proton} < DCS_{alpha}$*.*

Solution:

The constants and expressions to be used are the same as in the previous exercise, with the exception of $m_p c^2 = 938.272$ MeV and $z = 1$. As above, $\beta = 0.05426$, which corresponds to a proton energy of 1.384 MeV.

In this case the screening values are

$$\chi_0 = 3.544 \times 10^{-4} \text{ rad} \quad (0.020^\circ)$$
$$\chi_a = 7.313 \times 10^{-3} \text{ rad} \quad (0.419^\circ)$$

which when inserted in the expression for the screened DCS give

	Rutherford DCS (cm^2 rad^{-1})
θ(rad)	Screened
10^{-5}	2.3635×10^{-14}
10^{-4}	2.3626×10^{-14}
10^{-3}	2.2775×10^{-14}
10^{-2}	2.8701×10^{-15}
10^{-1}	6.7005×10^{-19}
10^{0}	7.9977×10^{-23}

Figure 2.2 compares the DCS for protons and for the α particles from the previous exercise. The figure shows that, above the screening angle, the DCS is larger for protons than for α particles, as expected from the lower proton rest energy. The larger screening angle for protons ($\sim 7.3 \times 10^{-3}$ rad) than for α particles ($\sim 4 \times 10^{-3}$ rad), due also to the $1/m_0c^2$ dependence of χ_0, cuts off the DCS earlier (when decreasing angle) than in the alpha case, and from that point downward, the DCS is smaller for protons than for the α particles whenever $\theta < \chi_a$.

3 (a) Derive an expression for the relativistic screened Rutherford total cross section (TCS) restricted to angles smaller than the atomic screening angle χ_a. (b) Compare with the unrestricted expression (Eq. (2.15)) for 5 MeV

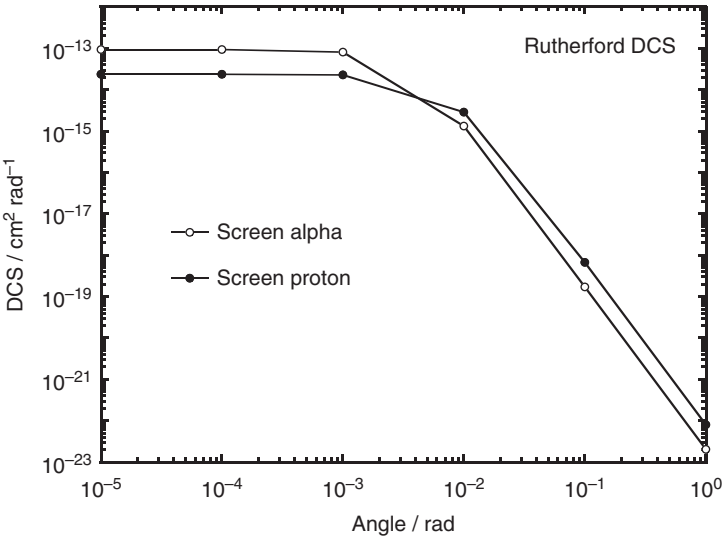


Figure 2.2 Screened Rutherford differential cross section for protons and α particles having the same speed β, incident on a 1 μm gold foil.

electrons incident on a mercury atom. (c) Calculate the mean free path of these electrons due to elastic collisions and the subsequent number of elastic collisions per path length.

Answer: (a) $\sigma_R[0, \chi_a] = (z\ Z\ r_e)^2 \left(\frac{m_e c^2}{m_0 c^2}\right)^2 \frac{1-\beta^2}{\beta^4} \frac{8\pi(1-\cos\chi_a)}{\chi_a^2(2+\chi_a^2-2\cos\chi_a)}$.
(b) $\sigma_R[0, \pi] = 2.10731 \times 10^{-18} cm^2$ *and* $\sigma_R[0, \chi_a] = 1.05366 \times 10^{-18} cm^2$, *that is, elastic collisions with* $\theta \leq \chi_a$ *account for practically 50% of the TCS. (c) MFP = 0.12 μm, that is, almost 10 elastic collisions occur per micron.*

Solution:

(a) The screened Rutherford DCS is given by

$$\frac{d\sigma_R}{d\Omega} = (z\ Z\ r_e)^2 \left(\frac{m_e c^2}{m_0 c^2}\right)^2 \frac{1-\beta^2}{\beta^4} \frac{1}{(1-\cos\theta + 0.5\chi_a^2)^2}$$

where χ_a is the screening angle, which in Molière's theory is given by

$$\chi_a^2 = \frac{4.2121 \times 10^{-3}\sqrt{1-\beta^2}}{m_0 c^2 \beta} Z^{1/3} \left[1.13 + 3.76\left(\frac{z\ Z}{137\beta}\right)^2\right]$$

The screened Rutherford TCS (Eq. (2.15)) was obtained from

$$\sigma_R[0, \pi] = \int_0^\pi \frac{d\sigma_R}{d\Omega} d\Omega = \int_0^\pi \frac{d\sigma_R}{d\Omega} 2\pi \sin\theta\ d\theta$$
$$= (z\ Z\ r_e)^2 \left(\frac{m_e c^2}{m_0 c^2}\right)^2 \frac{1-\beta^2}{\beta^4} \frac{16\ \pi}{\chi_a^2(4+\chi_a^2)}$$

Instead of integrating in the interval $[0, \pi]$, it is convenient to do it in the interval $[\theta_{\min}, \theta_{\max}]$, that is,

$$\sigma_{\mathrm{R}}[\theta_{\min}, \theta_{\max}] = \int_{\theta_{\min}}^{\theta_{\max}} \frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega} 2\pi \sin\theta \, \mathrm{d}\theta$$
$$= (z\, Z\, r_{\mathrm{e}})^2 \left(\frac{m_e c^2}{m_0 c^2}\right)^2 \frac{1-\beta^2}{\beta^4} 2\pi$$
$$\left[\frac{1}{1-\cos\theta_{\min}+0.5\chi_a^2} - \frac{1}{1-\cos\theta_{\max}+0.5\chi_a^2}\right]$$

which for $\theta_{\min} = 0$ and $\theta_{\max} = \pi$ coincides with the expression above. For the present exercise, we then take $\theta_{\min} = 0$ and $\theta_{\max} = \chi_a$ and get

$$\sigma_{\mathrm{R}}[0, \chi_a] = \int_0^{\chi_a} \frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega} 2\pi \sin\theta \, \mathrm{d}\theta$$
$$= (z\, Z\, r_{\mathrm{e}})^2 \left(\frac{m_{\mathrm{e}} c^2}{m_0 c^2}\right)^2 \frac{1-\beta^2}{\beta^4} \frac{8\pi(1-\cos\chi_a)}{\chi_a^2(2+\chi_a^2-2\cos\chi_a)}$$

(b) For 5 MeV electrons on mercury ($Z = 80$, $A = 200.59$, $\rho = 13.546$ g cm^{-3}), $\beta = 0.995692$, $m_0 c^2 = m_{\mathrm{e}} c^2$. Thus $\chi_a = 5.14873 \times 10^{-3}$ rad (0.3°). Note that we could have replaced Z^2 by $Z(Z+1)$. Therefore

$$\sigma_{\mathrm{R}}[0, \pi] = 2.10731 \times 10^{-18} \text{ cm}^2$$
$$\sigma_{\mathrm{R}}[0, \chi_a] = 1.05366 \times 10^{-18} \text{ cm}^2$$

that is, elastic collisions with $\theta \leq \chi_a$ account for practically 50% of the TCS.

The result above explains why even if we would have used the small-angle approximation,

$$\frac{\mathrm{d}\sigma_{\mathrm{R,small}}}{\mathrm{d}\Omega} = (z\, Z\, r_{\mathrm{e}})^2 \frac{1-\beta^2}{\beta^4} \frac{4}{(\theta^2+\chi_a^2)^2}$$

with corresponding TCS

$$\sigma_{\mathrm{R,small}}[\theta_{\min}, \theta_{\max}] = \int_{\theta_{\min}}^{\theta_{\max}} \frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}\Omega} 2\pi\theta \, \mathrm{d}\theta$$
$$= (z\, Z\, r_{\mathrm{e}})^2 \frac{1-\beta^2}{\beta^4} 4\pi \left[\frac{1}{\theta_{\min}^2+\chi_a^2} - \frac{1}{\theta_{\max}^2+\chi_a^2}\right]$$

that is, for $\theta_{\max} = 1$ rad and $\theta_{\max} = \chi_a$, we get, respectively,

$$\sigma_{\mathrm{R,small}}[0, 1] = (z\, Z\, r_{\mathrm{e}})^2 \frac{1-\beta^2}{\beta^4} \frac{4\pi}{\chi_a^2(1+\chi_a^2)}$$
$$\sigma_{\mathrm{R,small}}[0, \chi_a] = (z\, Z\, r_{\mathrm{e}})^2 \frac{1-\beta^2}{\beta^4} \frac{2\pi}{\chi_a^2},$$

the results would have been practically identical, as χ_a is very small and most collisions occur within $\theta \leq \chi_a$.