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Steven Holzner

Author, Physics For Dummies *and* Physics Workbook For Dummies

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by Steven Holzner

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Dedication

To Nancy, of course!

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Quantum Physics For Dummies®

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Introduction

Physics as a general discipline has no limits, from the very huge (galaxy-wide) to the very small (atoms and smaller). This book is about the very small side of things — that's the specialty of quantum physics. When you *quantize* something, you can't go any smaller; you're dealing with discrete units.

Classical physics is terrific at explaining things like heating cups of coffee or accelerating down ramps or cars colliding, as well as a million other things, but it has problems when things get very small. Quantum physics usually deals with the micro world, such as what happens when you look at individual electrons zipping around. For example, electrons can exhibit both particle and wave-like properties, much to the consternation of experimenters — and it took quantum physics to figure out the full picture.

Quantum physics also introduced the uncertainty principle, which says you can't know a particle's exact position and momentum at the same time. And the field explains the way that the energy levels of the electrons bound in atoms work. Figuring out those ideas all took quantum physics, as physicists probed ever deeper for a way to model reality. Those topics are all coming up in this book.

About This Book

Because uncertainty and probability are so important in quantum physics, you can't fully appreciate the subject without getting into calculus. This book presents the need-to-know concepts, but you don't see much in the way of thought experiments that deal with cats or parallel universes. I focus on the math and how it describes the quantum world.

I've taught physics to many thousands of students at the university level, and from that experience, I know most of them share one common trait: Confusion as to what they did to deserve such torture.

Quantum Physics For Dummies, Revised Edition largely maps to a college course, but this book is different from standard texts. Instead of writing it from the physicist's or professor's point of view, I've tried to write it from the reader's point of view. In other words, I've designed this book to be crammed full of the good stuff — and only the good stuff. Not only that, but you can discover ways of looking at things that professors and teachers use to make figuring out problems simple.

Although I encourage you to read this book from start to finish, you can also leaf through this book as you like, reading the topics that you find interesting. Like other *For Dummies* books, this one lets you skip around as you like as much as possible. You don't have to read the chapters in order if you don't want to. This is your book, and quantum physics is your oyster.

Conventions Used in This Book

Some books have a dozen dizzying conventions that you need to know before you can even start. Not this one. Here's all you need to know:

- I put new terms in italics, like *this,* the first time they're discussed; I follow them with a definition.

- Vectors — those items that have both a magnitude and a direction — are given in bold, like this: **B.**
- Web addresses appear in `monofont`.

Foolish Assumptions

I don't assume that you have any knowledge of quantum physics when you start to read this book. However, I do make the following assumptions:

- You're taking a college course in quantum physics, or you're interested in how math describes motion and energy on the atomic and subatomic scale.
- You have some math prowess. In particular, you know some calculus. You don't need to be a math pro, but you should know how to perform integration and deal with differential equations. Ideally, you also have some experience with Hilbert space.
- You have some physics background as well. You've had a year's worth of college-level physics (or understand all that's in *Physics For Dummies*) before you tackle this one.

How This Book Is Organized

Quantum physics — the study of very small objects — is actually a very big topic. To handle it, quantum physicists break the world down into different parts. Here are the various parts that are coming up in this book.

Part I: Small World, Huh? Essential Quantum Physics

Part I is where you start your quantum physics journey, and you get a good overview of the topic here. I survey quantum physics and tell you what it's good for and what kinds of problems it can solve. You also get a good foundation in the math that you need for the rest of the book, such as state vectors and quantum matrix manipulations. Knowing this stuff prepares you to handle the other parts.

Part II: Bound and Undetermined: Handling Particles in Bound States

Particles can be trapped inside potentials; for instance, electrons can be bound in an atom. Quantum physics excels at predicting the energy levels of particles bound in various potentials, and that's what Part II covers. You see how to handle particles bound in square wells and in harmonic oscillators.

Part III: Turning to Angular Momentum and Spin

Quantum physics lets you work with the micro world in terms of the angu-lar momentum of particles, as well as the spin of electrons. Many famous experiments — such as the Stern-Gerlach experiment, in which beams of particles split in magnetic fields — are understandable only in terms of quantum physics, and you get all the details here.

Part IV: Multiple Dimensions: Going 3D with Quantum Physics

In the first three parts, all the quantum physics problems are one-dimensional to make life a little easier while you're understanding how to solve those problems. In Part IV, you branch out to working with three-dimensional problems in both rectangular and spherical coordinate systems. Taking things from 1D to 3D gives you a better picture of what happens in the real world.

Part V: Group Dynamics: Introducing Multiple Particles

In this part, you work with multiple-particle systems, such as atoms and gases. You see how to handle many electrons in atoms, particles interacting with other particles, and particles that scatter off other particles.

Dealing with multiple particles is all another step in modeling reality — after all, systems with only a single particle don't take you very far in the real world, which is built of mega, mega systems of particles. In Part V, you see how quantum physics can handle the situation.

Part VI: The Part of Tens

You see the Part of the Tens in all *For Dummies* books. This part is made up of fast-paced lists of ten items each. You get to see some of the ten best online tutorials on quantum physics and a discussion of quantum physics' ten greatest triumphs.

Icons Used in This Book

You find a handful of icons in this book, and here's what they mean:

This icon flags particularly good advice, especially when you're solving problems.

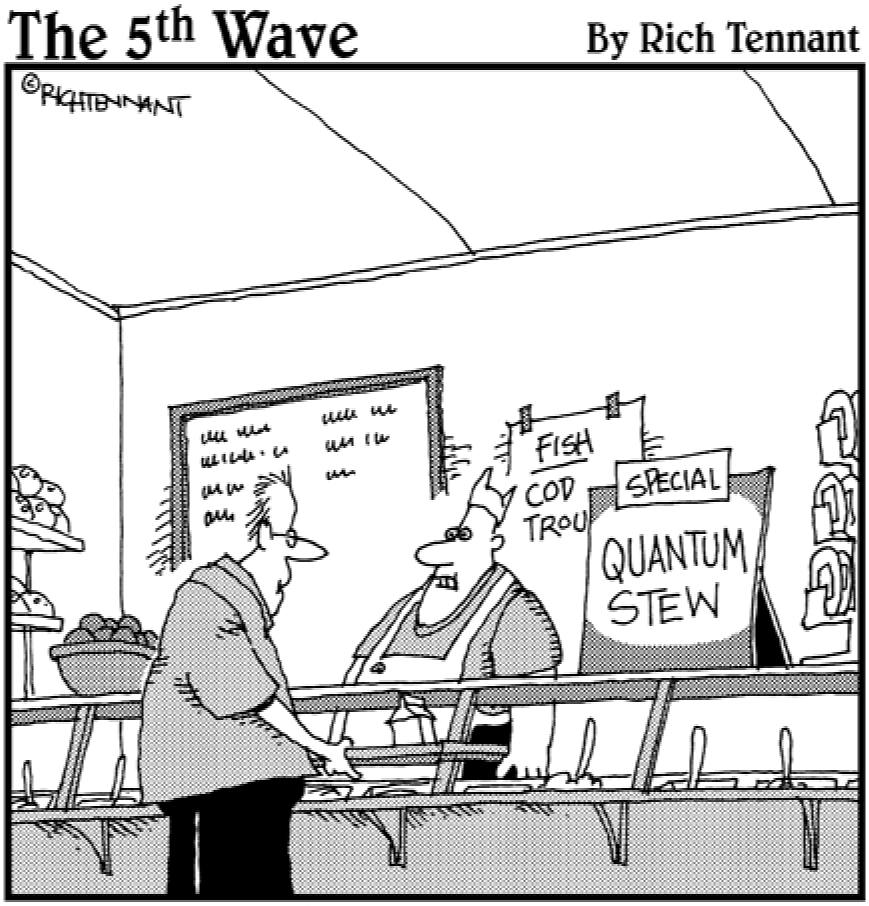
This icon marks something to remember, such as a law of physics or a particularly juicy equation.

This icon means that what follows is technical, insider stuff. You don't have to read it if you don't want to, but if you want to become a quantum physics pro (and who doesn't?), take a look.

This icon helps you avoid mathematical or conceptual slip-ups.

Where to Go from Here

All right, you're all set and ready to go. You can jump in anywhere you like. For instance, if you're sure electron spin is going to be a big topic of conversation at a party this weekend, check out Chapter 6. And if your upcoming vacation to Geneva, Switzerland, includes a side trip to your new favorite particle accelerator — the Large Hadron Collider — you can flip to Chapter 12 and read up on scattering theory. But if you want to get the full story from the beginning, jump into Chapter 1 first — that's where the action starts.



"It's just like the regular stew only it's got some bits of matter in it we can't identify."

Part I

Small World, Huh? Essential Quantum Physics

In this part . . .

This part is designed to give you an introduction to the ways of quantum physics. You see the issues that gave rise to quantum physics and the kinds of solutions it provides. I also introduce you to the kind of math that quantum physics requires, including the notion of state vectors.

Chapter 1

Discoveries and Essential Quantum Physics

In This Chapter

- Putting forth theories of quantization and discrete units
- Experimenting with waves acting as particles
- Experimenting with particles acting as waves
- Embracing uncertainty and probability

According to classical physics, particles are particles and waves are waves, and never the twain shall mix. That is, particles have an energy E and a momentum vector ***p,*** and that's the end of it. And waves, such as light waves, have an amplitude A and a wave vector ***k*** (where the magnitude of ***k*** = $\frac{2\pi}{\lambda}$, where λ is the wavelength) that points in the direction the wave is traveling. And that's the end of that, too, according to classical physics.

But the reality is different — particles turn out to exhibit wave-like properties, and waves exhibit particle-like properties as well. The idea that waves (like light) can act as particles (like electrons) and vice versa was the major revelation that ushered in quantum physics as such an important part of the world of physics. This chapter takes a look at the challenges facing classical physics around the turn of the 20th century — and how quantum physics gradually came to the rescue. Up to that point, the classical way of looking at physics was thought to explain just about everything. But as those pesky experimental physicists have a way of doing, they came up with a bunch of experiments that the theoretical physicists couldn't explain.

That made the theoretical physicists mad, and they got on the job. The problem here was the microscopic world — the world that's too tiny to see. On the larger scale, classical physics could still explain most of what was going on — but when it came to effects that depended on the micro-world, classical physics began to break down. Taking a look at how classical physics collapsed gives you an introduction to quantum physics that shows why people needed it.

Being Discrete: The Trouble with Black-Body Radiation

One of the major ideas of quantum physics is, well, quantization — measuring quantities in discrete, not continuous, units. The idea of quantized energies arose with one of the earliest challenges to classical physics: the problem of black-body radiation.

When you heat an object, it begins to glow. Even before the glow is visible, it's radiating in the infrared spectrum. The reason it glows is that as you heat it, the electrons on the surface of the material are agitated thermally, and electrons being accelerated and decelerated radiate light.

Physics in the late 19th and early 20th centuries was concerned with the spectrum of light being emitted by black bodies. A *black body* is a piece of material that radiates corresponding to its temperature — but it also absorbs and reflects light from its surroundings. To make matters easier, physics postulated a black body that reflected nothing and absorbed all the light falling on it (hence the term *black body*, because the object would appear perfectly black as it absorbed all light falling on it). When you heat a black body, it would radiate, emitting light.

Well, it was hard to come up with a physical black body — after all, what material absorbs light 100 percent and doesn't reflect anything? But the physicists were clever about this, and they came up with the hollow cavity you see in Figure 1-1, with a hole in it.

When you shine light on the hole, all that light would go inside, where it would be reflected again and again — until it got absorbed (a negligible amount of light would escape through the hole). And when you heated the hollow cavity, the hole would begin to glow. So there you have it — a pretty good approximation of a black body.

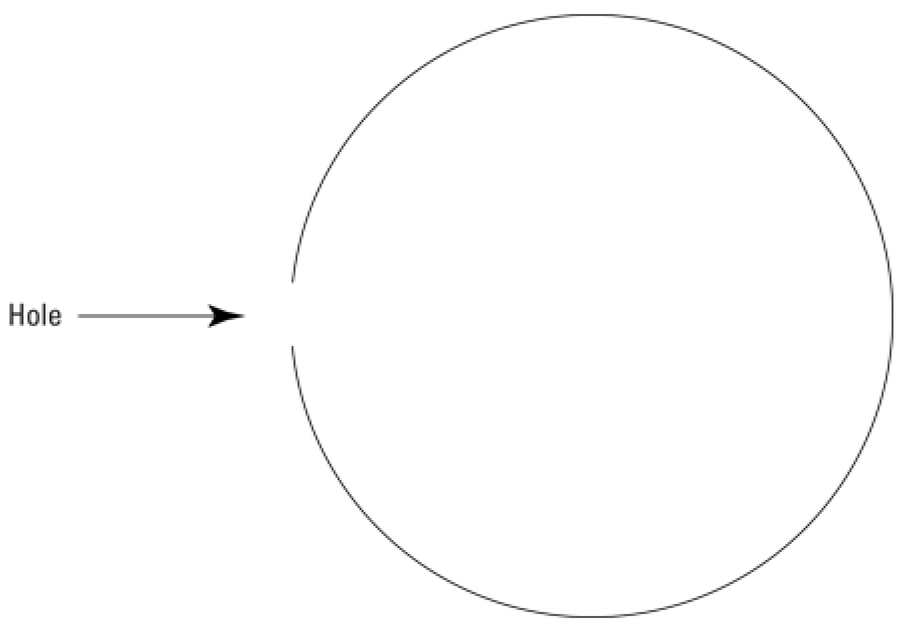


Figure 1-1: A black body.

You can see the spectrum of a black body (and attempts to model that spectrum) in Figure 1-2, for two different temperatures, T_1 and T_2. The problem was that nobody was able to come up with a theoretical explanation for the spectrum of light generated by the black body. Everything classical physics could come up with went wrong.

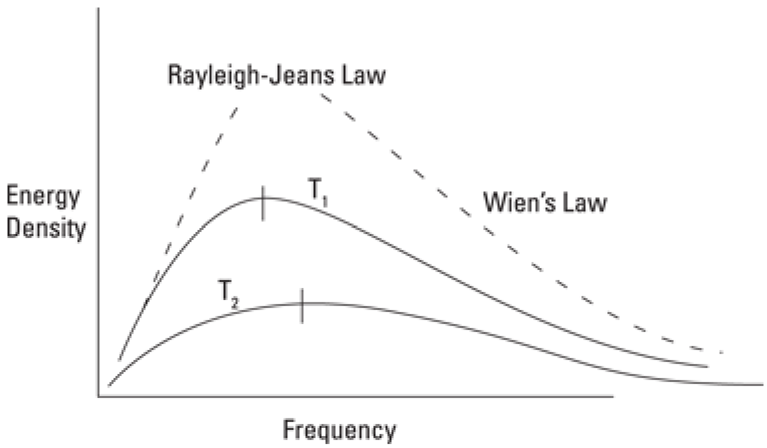


Figure 1-2: Black-body radiation spectrum.

First attempt: Wien's Formula

The first one to try to explain the spectrum of a black body was Wilhelm Wien, in 1889. Using classical thermodynamics, he came up with this formula:

$$u(\upsilon, T) = A\upsilon^5 e^{-\beta\upsilon/T}$$

where *u* (υ, T) is the intensity distribution of the light spectrum at frequency υ of a black body at the temperature T, and A and β are constants which can be measured in experiments. (The spectrum is given by *u*[υ, T], which is the energy density of the emitted light as a function of frequency and temperature.)

This equation, Wien's formula, worked fine for high frequencies, as you can see in Figure 1-2; however, it failed for low frequencies.

Second attempt: Rayleigh-Jeans Law

Next up in the attempt to explain the black-body spectrum was the Rayleigh-Jeans Law, introduced around 1900. This law predicted that the spectrum of a black body was

$$u(\upsilon,\mathrm{T}) = \frac{2\pi\upsilon^4}{c^3}k\mathrm{T}$$

where k is Boltzmann’s constant (approximately 1.3807 × 10–23 J·K–1). However, the Rayleigh-Jeans Law had the opposite problem of Wien’s law: Although it worked well at low frequencies (see Figure 1-2), it didn’t match the higher-frequency data at all — in fact, it diverged at higher frequencies. This was called the *ultraviolet catastrophe* because the best predictions available diverged at high frequencies (corresponding to ultraviolet light). It was time for quantum physics to take over.

An intuitive (quantum) leap: Max Planck’s spectrum

The black-body problem was a tough one to solve, and with it came the first begin-nings of quantum physics. Max Planck came up with a radical suggestion — what if the amount of energy that a light wave can exchange with matter wasn’t continuous, as postulated by classical physics, but *discrete?* In other words, Planck postulated that the energy of the light emitted from the walls of the black-body cavity came only in integer multiples like this, where *h* is a universal constant:

$$\mathrm{E} = nh\upsilon, \text{ where } n = 0, 1, 2, \ldots$$

With this theory, crazy as it sounded in the early 1900s, Planck converted the continuous integrals used by Rayleigh-Jeans to discrete sums over an infinite number of terms. Making that simple change gave Planck the