

Counterfactuals

David Lewis



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Counterfactuals

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BLACKWELL PUBLISHING

350 Main Street, Malden, MA 02148-5020, USA

108 Cowley Road, Oxford OX4 1JF, UK

550 Swanston Street, Carlton, Victoria 3053, Australia

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First published by Basil Blackwell Ltd 1973

First publishing in the USA by Harvard University Press 1973

Reissued by Blackwell Publishers 2001

Reprinted 2003, 2005

Library of Congress Cataloging-in-Publication Data

Lewis, David K., 1941-

Counterfactuals / David Lewis.

p. cm.

First published: 1973.

Includes bibliographical references and index.

ISBN 0-631-22495-5 (hardback.: alk. paper) — ISBN 0-631-22425-4 (pbk. : alk. paper)

1. Counterfactuals (Logic) I. Title.

BC199.C66 L48 2000

160—dc21

00-059899

A catalogue record for this title is available from the British Library.

The publisher's policy is to use permanent paper from mills that operate a sustainable forestry policy, and which has been manufactured from pulp processed using acid-free and elementary chlorine-free practices. Furthermore, the publisher ensures that the text paper and cover board used have met acceptable environmental accreditation standards.

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IN MEMORY OF RICHARD MONTAGUE

Preface

The principal changes in this revised printing are in Section 6.1, where I have corrected two major errors in my discussion of completeness results for the *V*-logics. Both of them were spotted by Erik C. W. Krabbe in 1976. I am most grateful to him for finding the trouble, and also for very helpful correspondence about alternative methods of repair. One error was in my construction of the canonical basis on pages 127–130: I falsely claimed that the set of co-spheres of cuts around a given index would be closed under unions.* In order to ensure such closure, it is necessary to construct the canonical basis differently. The other was in the axiom system for VC given on page 132. I left out the rule of Interchange of Logical Equivalents; however I tacitly appealed to this rule in proving completeness, so my proof did not apply to the axiom system I had given.

In addition I have corrected minor errors on pages 35, 55 and 129, also spotted by Krabbe; removed misprints; and brought some references up to date.

I have had more to say about counterfactuals and related matters. These further thoughts might appropriately have been added to this book; but since they are to be found elsewhere, I have been content to add an appendix giving citations and abstracts.

David Lewis
1986

* Erik C. W. Krabbe, 'Note on a Completeness Theorem in the Theory of Counterfactuals', *Journal of Philosophical Logic* **7** (1978): 91–93.

Acknowledgements

I am grateful to Kit Fine, Hans Kamp, David Kaplan, Richard Montague, J. Howard Sobel, Robert Stalnaker, Richmond Thomason, and many other friends and colleagues for encouragement and for valuable discussions about counterfactuals over the last five years.

I am grateful also to the American Council of Learned Societies for financial assistance, and to Saint Catherine's College, Oxford, for hospitality, during the year when most of this book was written.

David Lewis

Princeton, June 1972

1. An Analysis of Counterfactuals

1.1 Introduction

'If kangaroos had no tails, they would topple over' seems to me to mean something like this: in any possible state of affairs in which kangaroos have no tails, and which resembles our actual state of affairs as much as kangaroos having no tails permits it to, the kangaroos topple over. I shall give a general analysis of counterfactual conditionals along these lines.

My methods are those of much recent work in possible-world semantics for intensional logic.* I shall introduce a pair of counterfactual conditional operators intended to correspond to the various counterfactual conditional constructions of ordinary language; and I shall interpret these operators by saying how the truth value at a given possible world of a counterfactual conditional is to depend on the truth values at various possible worlds of its antecedent and consequent.

Counterfactuals are notoriously vague. That does not mean that we cannot give a clear account of their truth conditions. It does mean that such an account must either be stated in vague terms—which does *not* mean ill-understood terms—or be made relative to some parameter that is fixed only within rough limits on any given occasion of language use. It is to be hoped that this imperfectly fixed parameter is a familiar one that we would be stuck with whether or not we used it in the analysis of counterfactuals; and so it will be. It will be a relation of comparative similarity.

Let us employ a language containing these two counterfactual conditional operators:

$\Box \rightarrow$

read as 'If it were the case that ___, then it would be the case that...', and

$\Diamond \rightarrow$

read as 'If it were the case that ___, then it might be the case that...'. For instance, the two sentences below would be symbolized as shown.

If Otto behaved himself, he would be ignored.

Otto behaves himself $\Box \rightarrow$ Otto is ignored

If Otto were ignored, he might behave himself.

Otto is ignored $\Diamond \rightarrow$ Otto behaves himself

There is to be no prohibition against embedding counterfactual conditionals within other counterfactual conditionals. A sentence of such a form as this.

$$((\psi \Box \rightarrow ((\chi \Box \rightarrow \psi) \Diamond \rightarrow \phi)) \Diamond \rightarrow \chi) \\ \Box \rightarrow (\phi \Box \rightarrow (\psi \Diamond \rightarrow ((\chi \Box \rightarrow \phi) \Diamond \rightarrow (\phi \Box \rightarrow \psi))))$$

will be perfectly well formed and will be assigned truth conditions, although doubtless it would be such a confusing sentence that we never would have occasion to utter it.

The two counterfactual operators are to be interdefinable as follows.

$$\phi \Diamond \rightarrow \psi =_{df} \sim(\phi \Box \rightarrow \sim\psi), \\ \phi \Box \rightarrow \psi =_{df} \sim(\phi \Diamond \rightarrow \sim\psi).$$

Thus we can take either one as primitive. Its interpretation determines the interpretation of the other. I shall take the 'would' counterfactual $\Box \rightarrow$ as primitive.

Other operators can be introduced into our language by definition in terms of the counterfactual operators, and it will prove useful to do so. Certain modal operators will be thus introduced in Sections 1.5 and 1.7; modified versions of

the counterfactual in Section 1.6; and 'comparative possibility' operators in Section 2.5.

My official English readings of my counterfactual operators must be taken with a good deal of caution. First, I do not intend that they should interfere, as the counterfactual constructions of English sometimes do, with the tenses of the antecedent and consequent. My official reading of the sentence

We were finished packing Monday night $\square \rightarrow$ we departed Tuesday morning

comes out as a sentence obscure in meaning and of doubtful grammaticality:

If it were the case that we were finished packing Monday night, then it would be the case that we departed Tuesday morning.

In the correct reading, the subjunctive 'were' of the counterfactual construction and the temporal 'were' of the antecedent are transformationally combined into a past subjunctive:

If we had been finished packing Monday night, then we would have departed Tuesday morning.

Second, the ‘If it were the case that___’ of my official reading of $\Box \rightarrow$ is not meant to imply that it is not the case that___. Counterfactuals with true antecedents—counterfactuals that are not counterfactual—are not automatically false, nor do they lack truth value. This stipulation does not seem to me at all artificial. Granted, the counterfactual constructions of English do carry some sort of presupposition that the antecedent is false. It is some sort of mistake to use them unless the speaker does take the antecedent to be false, and some sort of mishap to use them when the speaker wrongly takes the antecedent to be false. But there is no reason to suppose that every sort of presupposition failure must produce automatic falsity or a truth-value gap. Some or all sorts of presupposition, and in particular the presupposition that the antecedent of a counterfactual is false, may be mere matters of conversational implicature, without any effect on truth conditions. Though it is difficult to find out the truth conditions of counterfactuals with true antecedents, since they would be asserted only by mistake, we will see later (in Section 1.7) how this may be done.

You may justly complain, therefore, that my title ‘Counterfactuals’ is too narrow for my subject. I agree, but I know no better. I cannot claim to be giving a theory of conditionals in general. As Ernest Adams has observed,^{*} the first conditional below is probably true, but the second may very well be false. (Change the example if you are not a Warrenite.)

If Oswald did not kill Kennedy, then someone else did.

If Oswald had not killed Kennedy, then someone else would have.

Therefore there really are two different sorts of conditional; not a single conditional that can appear as indicative or as counterfactual depending on the speaker’s opinion about the truth of the antecedent.

The title ‘Subjunctive Conditionals’ would not have delineated my subject properly. For one thing, there are shortened counterfactual conditionals like ‘*No Hitler, no A-bomb*’ that have no subjunctives except in their—still all-too-hypothetical—deep structure. More important, there are subjunctive conditionals pertaining to the future, like ‘*If our ground troops entered Laos next year, there would be trouble*’ that appear to have the truth conditions of indicative conditionals, rather than of the counterfactual conditionals I shall be considering.*

1.2 Strict Conditionals

We shall see that the counterfactual cannot be any strict conditional. Since it turns out to be something not too different, however, let us set the stage by reviewing the interpretation of strict conditionals in the usual possible-world semantics for modality. Generally speaking, a strict conditional is a material conditional preceded by some sort of necessity operator:

$$\Box(\phi \supset \psi).$$

With every necessity operator \Box there is paired its dual possibility operator \Diamond . The two are interdefinable:

$$\Diamond\phi =_{\text{df}} \sim\Box\sim\phi, \text{ or } \Box\phi =_{\text{df}} \sim\Diamond\sim\phi.$$

If we like, we can rewrite the strict conditional using the possibility operator:

$$\sim\Diamond(\phi \ \& \ \sim\psi).$$

Or we could introduce a primitive strict conditional arrow or hook, and define the necessity and possibility operators from that. ‡

A *necessity operator*, in general, is an operator that acts like a restricted universal quantifier over possible worlds. Necessity of a certain sort is truth at all possible worlds that

satisfy a certain restriction. We call these worlds *accessible*, meaning thereby simply that they satisfy the restriction associated with the sort of necessity under consideration. Necessity is truth at all accessible worlds, and different sorts of necessity correspond to different accessibility restrictions. A *possibility operator*, likewise, is an operator that acts like a restricted existential quantifier over worlds. Possibility is truth at some accessible world, and the accessibility restriction imposed depends on the sort of possibility under consideration. If a necessity operator and a possibility operator correspond to the same accessibility restriction on the worlds quantified over, then they will be a dual, interdefinable pair.

In the case of *physical necessity*, for instance, we have this restriction: the accessible worlds are those where the actual laws of nature hold true. Physical necessity is truth at all worlds where those laws hold true; physical possibility is truth at some worlds where those laws hold true.

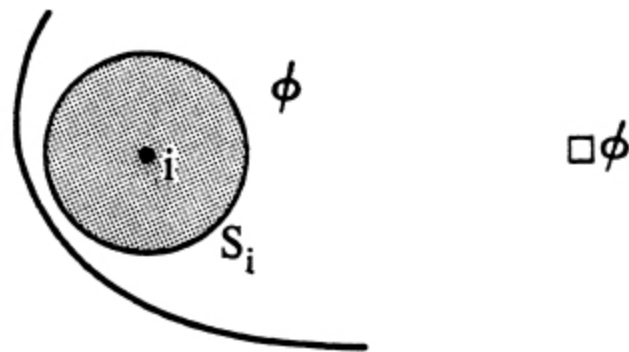
In the case of physical necessity, which possible worlds are admitted as accessible depends on what the actual laws of nature happen to be. The restriction will be different from the standpoint of worlds with different laws of nature. Let i and j be worlds with different laws of nature, and let k be a world where the laws of i hold true but the different laws of j are violated. From the standpoint of i , k is an accessible world; from the standpoint of j it is not. Accessibility is in this case—and most cases—a relative matter. It is the custom, therefore, to think of accessibility as a relation between worlds: we say that k is *accessible from* i , but k is not accessible from j . We say also that i stands to k , but j does not stand to k , in the *accessibility relation* for physical necessity and possibility.

In general: to a necessity operator \Box or a possibility operator \Diamond there corresponds an accessibility relation. The appropriate accessibility relation serves to restrict

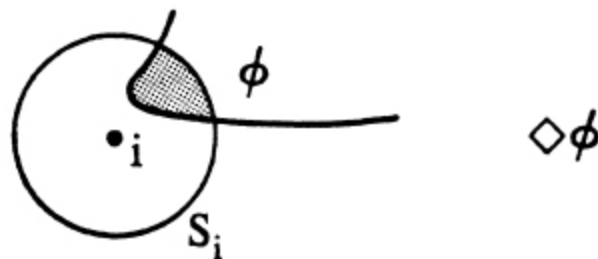
quantification over worlds in giving the truth conditions for \Box or \Diamond . For any possible world i and sentence ϕ , the sentence $\Box\phi$ is true at the world i if and only if, for every world j such that j is accessible from i , ϕ is true at j . Likewise $\Diamond\phi$ is true at i if and only if, for some world j such that j is accessible from i , ϕ is true at j . More concisely: $\Box\phi$ is true at i if and only if ϕ is true at every world accessible from i ; $\Diamond\phi$ is true at i if and only if ϕ is true at some world accessible from i . It follows that the strict conditional $\Box(\phi \supset \psi)$ is true at i if and only if, for every world j such that j is accessible from i , the material conditional $\phi \supset \psi$ is true at j ; that is, if and only if, for every world j such that j is accessible from i and ϕ is true at j , ψ is true at j . More concisely: $\Box(\phi \supset \psi)$ is true at i if and only if ψ is true at every accessible ϕ -world. (' ϕ -world', of course, abbreviates 'world at which ϕ is true', and likewise for parallel formations.)

FIGURE 1

(A) NECESSITY



(B) POSSIBILITY



(C) STRICT CONDITIONAL



It suits my purposes better not to use the customary accessibility relations, but instead to adopt a slightly different—but obviously equivalent—formulation. Corresponding to a necessity operator \Box , or a possibility operator \Diamond , or a kind of strict conditional, let us have an assignment to each world i of a set S_i of worlds, called the *sphere of accessibility* around i and regarded as the set of worlds accessible from i .^{*} The assignment of spheres to worlds may be called the *accessibility assignment*

corresponding to the modal operator. It is used to give the truth conditions for modal sentences as follows.

A sentence $\Box\phi$ is true at a world i if and only if ϕ is true throughout the sphere of accessibility S_i around i (as shown in [Figure 1\(A\)](#)).

A sentence $\Diamond\phi$ is true at a world i if and only if ϕ is true somewhere in the sphere S_i (as shown in [Figure 1\(B\)](#)).

A strict conditional sentence $\Box(\phi \supset \psi)$ is true at i if and only if $\phi \supset \psi$ is true throughout the sphere S_i ; that is, if and only if ψ is true at every ϕ -world in S_i (as shown in [Figure 1\(C\)](#)).

Let us consider various examples of accessibility assignments for various sorts of necessity, with particular attention to the corresponding strict conditionals.

Corresponding to *logical necessity*, and the logical strict conditional, we assign to each world i as its sphere of accessibility S_i the set of *all* possible worlds. Thus the logical strict conditional $\Box(\phi \supset \psi)$ is true at i if and only if ψ is true at all ϕ -worlds whatever; there are no inaccessible ϕ -worlds to be left out of consideration.

Corresponding to *physical necessity*, and the physical strict conditional, we assign to each world i as its sphere of accessibility S_i the set of all worlds where the laws of nature prevailing at i hold; so the physical strict conditional $\Box(\phi \supset \psi)$ is true at i if and only if ψ is true at all those ϕ -worlds where the laws prevailing at i hold.

Corresponding to a kind of time-dependent necessity we may call *inevitability at time t* , and its strict conditional, we assign to each world i as its sphere of accessibility the set of all worlds that are exactly like i at all times up to time t , so $\Box(\phi \supset \psi)$ is true at i if and only if ψ is true at all ϕ -worlds that are exactly like i up to t .

Corresponding to what we might call *necessity in respect of facts of so-and-so kind*, and its strict conditional, we

assign to each world i as its sphere of accessibility the set of all worlds that are exactly like i in respect of all facts of so-and-so kind, so $\Box(\phi \supset \psi)$ is true at i if and only if ψ is true at all ϕ -worlds that are exactly like i in respect of all facts of so-and-so kind.

A degenerate case: corresponding to what we may call *necessity in respect of all facts*, or *fatalistic necessity*, we assign to each world i as its sphere of accessibility the set of all worlds that are exactly like i in all respects whatever. Since 'all respects whatever' includes likeness in respect of identity or nonidentity to i , i alone is like i in all respects whatever; thus each world i has as its sphere of accessibility the set $\{i\}$ having i as its sole member. Then $\Box\phi$ is true at i if and only if ϕ is true at i ; and the fatalistic strict conditional $\Box(\phi \supset \psi)$ is true at i if and only if the material conditional $\phi \supset \psi$ is true at i .

Sometimes we do not insist that each world i must belong to its own sphere of accessibility S_i . Corresponding to *deontic* (or moral) necessity, we assign to each world i as its sphere of accessibility the set of all morally perfect worlds. Then $\Box\phi$ is true at i if and only if ϕ is true at every morally perfect world. A morally imperfect world like ours does not belong to its own sphere of accessibility.

We have another degenerate case: corresponding to what I may call *vacuous necessity*, we assign to each world i as its sphere of accessibility the empty set, making $\Box\phi$ true at i for any sentence ϕ and world i whatever.

We may compare the strictness of different strict conditionals. The more inclusive are the spheres of accessibility, the stricter is the conditional. Suppose we have necessity operators \Box_1 and \Box_2 , corresponding to the assignment to each world i of spheres of accessibility S^1_i and S^2_i respectively. Then the strict conditional $\Box_2(\phi \supset \psi)$ is *stricter at world i* than $\Box_1(\phi \supset \psi)$ if and only if S^2_i properly

includes S ? One strict conditional is *stricter* than another if and only if the first is stricter at every world. Note that any strict conditional is implied by any stricter conditional with the same antecedent and consequent.

Thus the logical strict conditional is stricter than any other; the material conditional is the least strict of all the conditionals that obey the constraint that every world is self-accessible; and the physical strict conditional, for instance, falls in between. The vacuous conditional is the least strict conditional of all.

It may happen, of course, that two strict conditionals are incomparable. It may be that they are incomparable at some world because neither sphere includes the other. Or they may be comparable at every world, but one may be stricter at some worlds and the other at other worlds.

Counterfactuals are related to a kind of strict conditional based on comparative similarity of possible worlds. A counterfactual $\phi \Box \rightarrow \psi$ is true at a world i if and only if ψ holds at certain ϕ -worlds; but certainly not all ϕ -worlds matter. '*If kangaroos had no tails, they would topple over*' is true (or false, as the case may be) at our world, quite without regard to those possible worlds where kangaroos walk around on crutches, and stay upright that way. Those worlds are too far away from ours. What is meant by the counterfactual is that, things being pretty much as they are—the scarcity of crutches for kangaroos being pretty much as it actually is, the kangaroos' inability to use crutches being pretty much as it actually is, and so on—if kangaroos had no tails they would topple over.

We might think it best to confine our attention to worlds where kangaroos have no tails and *everything* else is as it actually is; but there are no such worlds. Are we to suppose that kangaroos have no tails but that their tracks in the sand are as they actually are? Then we shall have to suppose that these tracks are produced in a way quite

different from the actual way. Are we to suppose that kangaroos have no tails but that their genetic makeup is as it actually is? Then we shall have to suppose that genes control growth in a way quite different from the actual way (or else that there is something, unlike anything there actually is, that removes the tails). And so it goes; respects of similarity and difference trade off. If we try too hard for exact similarity to the actual world in one respect, we will get excessive differences in some other respect.

There is a simpler argument that there is no world where kangaroos have no tails and everything else is as it actually is. Consider all the material conditionals of the form

$\phi \supset \textit{kangaroos have tails}$

such that ϕ is true at the actual world. If kangaroos had no tails and everything else were as it actually is, then these conditionals would be true as they actually are, for these conditionals are part of the 'everything else'. Also, in most cases, the antecedents would be true as they actually are, for (at least when the antecedent is irrelevant to whether kangaroos have tails) the antecedents also are part of the 'everything else'. But then, unless the world is one where *modus ponens* goes haywire (so that logic itself is not as it actually is!), kangaroos do have tails there after all. I know of nothing wrong with this argument, but I admit that it looks like an unconvincing trick; so I prefer to rely on the considerations of the previous paragraph.

It therefore seems as if counterfactuals are strict conditionals corresponding to an accessibility assignment determined by similarity of worlds—overall similarity, with respects of difference balanced off somehow against respects of similarity. Let S_i , for each world i , be the set of all worlds that are similar to at least a certain fixed degree to the world i . Then the corresponding strict conditional is true at i if and only if the material conditional of its antecedent and consequent is true throughout S_i ; that is, if

and only if the consequent holds at all antecedent-worlds similar to at least that degree to *i*.

If we take any one counterfactual, this will do nicely. But trouble may come if we consider several counterfactuals together. (1) *'If I (or you, or anyone else) walked on the lawn, no harm at all would come of it; but if everyone did that, the lawn would be ruined'* (2) *'If the USA threw its weapons into the sea tomorrow, there would be war; but if the USA and the other nuclear powers all threw their weapons into the sea tomorrow there would be peace; but if they did so without sufficient precautions against polluting the world's fisheries there would be war; but if, after doing so, they immediately offered generous reparations for the pollution there would be peace; ...'*^{*} (3) *'If Otto had come, it would have been a lively party; but if both Otto and Anna had come it would have been a dreary party; but if Waldo had come as well, it would have been lively; but. ...'*

These sequences have the following general form. I include with each asserted counterfactual also the negated opposite, for in the cases I imagine these negated opposites also are held true.

$$\begin{array}{ll}
 \phi_1 \Box \rightarrow \psi & \text{and } \sim(\phi_1 \Box \rightarrow \sim\psi), \\
 \phi_1 \& \phi_2 \Box \rightarrow \sim\psi & \text{and } \sim(\phi_1 \& \phi_2 \Box \rightarrow \psi), \\
 \phi_1 \& \phi_2 \& \phi_3 \Box \rightarrow \psi & \text{and } \sim(\phi_1 \& \phi_2 \& \phi_3 \Box \rightarrow \sim\psi), \\
 \vdots &
 \end{array}$$

With a little ingenuity, it seems possible to prolong such a sequence indefinitely. No one stage in the sequence refutes the theory that the counterfactual is a strict conditional based on similarity, but any two adjacent stages do. The counterfactual on the left at any stage contradicts the negated counterfactual on the right at the next stage. Take the first and second stages: no matter how the spheres of accessibility may be assigned, if ψ is true at every accessible ϕ_1 -world, then ψ is true at every accessible $(\phi_1 \& \phi_2)$ -world. So if the counterfactual is any strict conditional

whatever, then $\phi_1 \Box \rightarrow \psi$ implies $\phi_1 \ \& \ \phi_2 \Box \rightarrow \psi$ and contradicts $\sim (\phi_1 \ \& \ \phi_2 \Box \rightarrow \psi)$. Likewise $\phi_1 \ \& \ \phi_2 \Box \rightarrow \sim \psi$ implies $\phi_1 \ \& \ \phi_2 \ \& \ \phi_3 \Box \rightarrow \sim \psi$ and contradicts $\sim (\phi_1 \ \& \ \phi_2 \ \& \ \phi_3 \Box \rightarrow \sim \psi)$, and so on down the sequence.

The left-hand counterfactuals make trouble for the theory that the counterfactual is a strict conditional, even without their negated opposites. If those at two adjacent stages both are true, then according to the theory the second is true vacuously. So are all those beyond it. Beginning at the beginning: if ψ is true at every accessible ϕ_1 -world but $\sim \psi$ is true at every accessible $(\phi_1 \ \& \ \phi_2)$ -world, then there must not be any accessible $(\phi_1 \ \& \ \phi_2)$ -worlds—nor any accessible $(\phi_1 \ \& \ \phi_2 \ \& \ \phi_3)$ -worlds, nor. ... Then if the lower counterfactuals are true, it is no thanks to their consequents: if a strict conditional is vacuously true, then so is any other with the same antecedent. From the premises that if Otto had come it would have been lively and that if Otto and Anna had come it would have been dreary, it follows that if Otto and Anna had come then the cow would have jumped over the moon. Since that does *not* follow, the counterfactual is not a strict conditional.

If we treat the counterfactual as a strict conditional based on similarity, then the best we can do for our troublesome sequences is to keep changing our minds about which such strict conditional it is. We may be able to make the two sentences at any one stage true by an appropriate choice of a sphere of accessibility based on similarity, but we must choose anew for each stage. If so, we have the situation shown in [Figure 2](#). Suppose we have a sphere S_i^1 around i that is right for the first stage: ψ is true at every ϕ_1 -world in S_i^1 , and—since there are ϕ_1 -worlds in S_i^1 —it is not the case that $\sim \psi$ also is true at every ϕ_1 -world in S_i^1 . Then S_i^1 is wrong for the second stage. So is any sphere smaller than S_i^1 . But