

Paul I. Palmer

$$4x \div 5, 6 = 2 \frac{3}{4}, \div 7, \div 8$$

$$3(x + y + 4) = 3x + 3y + 12$$

$$2x + 4x + 6x + 8x = 20x$$

$$2x \div 7, \frac{3}{4}$$



Essential Maths for Geoscientists

An Introduction



WILEY Blackwell

Essential Maths for Geoscientists

An Introduction

Paul I. Palmer

School of Geosciences, University of Edinburgh, UK

WILEY Blackwell

This edition first published 2014 © 2014 by John Wiley & Sons, Ltd

Registered office: John Wiley & Sons, Ltd, The Atrium, Southern Gate,
Chichester, West Sussex,
PO19 8SQ, UK

Editorial offices: 9600 Garsington Road, Oxford, OX4 2DQ, UK
The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK
111 River Street, Hoboken, NJ 07030-5774, USA

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com/wiley-blackwell.

The right of the author to be identified as the author of this work has been asserted in accordance with the UK Copyright, Designs and Patents Act 1988.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by the UK Copyright, Designs and Patents Act 1988, without the prior permission of the publisher.

Designations used by companies to distinguish their products are often claimed as trademarks. All brand names and product names used in this book are trade names, service marks, trademarks or registered trademarks of their respective owners. The publisher is not associated with any product or vendor mentioned in this book.

Limit of Liability/Disclaimer of Warranty: While the publisher and author(s) have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. It is sold on the understanding that the publisher is not engaged in rendering professional services and neither the publisher nor the author shall be liable for damages arising herefrom. If professional advice or other expert assistance is required, the services of a competent professional should be sought.

Library of Congress Cataloging-in-Publication Data

Palmer, Paul I.

Essential maths for geoscientists : an introduction / Paul I. Palmer.

pages cm

Includes bibliographical references and index.

ISBN 978-0-470-97193-2 (cloth) – ISBN 978-0-470-97194-9 (pbk.) 1.
Geology-Mathematics. 2. Mathematics-Study and teaching. 3. Ecology-
Mathematical models. 4. Environmental protection-Mathematical models. I.
Title. II. Title: Essential math for geoscientists.

QE33.2.M3P35 2014

510.24'55-dc23

2013044549

A catalogue record for this book is available from the British Library.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books.

To A.L.M., L.M.P., and J.S.P.

CONTENTS

[Preface](#)

[1 How Do You Know that Global Warming Is Not a Hoax?](#)

[The Earth system: how do we know what we know?](#)

[Notes](#)

[2 Preamble:](#)

[2.1 The scientific method: pushing back the frontiers of ignorance](#)

[2.2 Subscript and superscripts](#)

[2.3 Scientific number format](#)

[2.4 Significant figures and rounding numbers](#)

[2.5 Units and dimensions](#)

[2.6 Symbols and numbers](#)

[2.7 Mean, median and variance: commonly encountered statistics](#)

[2.8 Guesstimation](#)

[Notes](#)

[2.9 Exercises](#)

[3 Algebra](#)

[3.1 Introduction](#)

[3.2 Evaluating algebraic equations](#)

[3.3 Simplifying algebraic equations](#)

[3.4 Factorization](#)

[3.5 Transposing formulae](#)

[3.6 Word problems](#)

[3.7 Exercises](#)

[4 Solving Equations](#)

[4.1 Solving linear equations](#)

[4.2 Solving simultaneous equations](#)

[4.3 Solving quadratic equations](#)

[4.4 Exercises](#)

[5 Logarithms and Exponentials](#)

[5.1 Exponentials](#)

[5.2 Logarithms](#)

[5.3 Log-normal and log-log plots: when and how to use them](#)

[Notes](#)

[5.4 Exercises](#)

[6 Uncertainties, Errors, and Statistics](#)

[6.1 Errors](#)

[6.2 Combining errors](#)

[6.3 Statistics](#)

[6.4 Correlations](#)

[Notes](#)

[6.5 Exercises](#)

[7 Trigonometry](#)

[7.1 Some geoscience applications of trigonometry](#)

[7.2 Anatomy of a triangle](#)

[7.3 Angles: degrees and radians](#)

[7.4 Calculating angles given a trigonometric ratio](#)

[7.5 Cosine and sine rules for non-right-angled triangles](#)

[7.6 Exercises](#)

[8 Vectors](#)

[8.1 What is a vector?](#)

[8.2 Resolving a vector](#)

[8.3 Vector algebra](#)

[8.4 Resolving non-perpendicular vectors](#)

[8.5 Exercises](#)

[9 Calculus 1: Differentiation:](#)

[9.1 A graphical interpretation of differentiation](#)

[9.2 A general formula for differentiation](#)

[9.3 The derivative of some common functions](#)

[9.4 Differentiation of the sum and difference of functions](#)

[9.5 Higher derivatives](#)

[9.6 Maxima and minima](#)

[Notes](#)

[9.7 Exercises](#)

[10 Calculus 2: Integration](#)

[10.1 Introduction](#)

[10.2 Definite integrals](#)

[10.3 Numerical integration](#)

[10.4 Exercises](#)

[11 Bringing It All Together](#)

[A Answers to Problems](#)

[A.1 Chapter 2: Preamble](#)

[A.2 Chapter 3: Algebra](#)

[A.3 Chapter 4: Solving Equations](#)

[A.4 Chapter 5: Logarithms and Exponentials](#)

[A.5 Chapter 6: Uncertainties, Errors, and Statistics](#)

[A.6 Chapter 7: Trigonometry](#)

[A.7 Chapter 8: Vectors](#)

[A.8 Chapter 9: Differentiation](#)

[A.9 Chapter 10: Integration](#)

[A.10 Chapter 11: Bringing it all together](#)

[B A Brief Note on Excel:](#)

[C Further Reading](#)

[Index](#)

List of Tables

[Chapter 2](#)

[**Table 2.1**](#)

[**Table 2.2**](#)

[**Table 2.3**](#)

[**Table 2.4**](#)

[Chapter 5](#)

[Table 5.1](#)

[Table 5.2](#)

[Chapter 6](#)

[Table 6.1](#)

[Chapter 8](#)

[Table 8.1](#)

[Table 8.2](#)

[Table 8.3](#)

[Chapter 9](#)

[Table 9.1](#)

[Chapter 11](#)

[Table 11.1](#)

[Table 11.2](#)

[Table 11.3](#)

[Table 11.4](#)

[Table 11.5](#)

[Table 11.6](#)

[A Answers to Problems](#)

[**Table A.1**](#)

[**Table A.2**](#)

[**Table A.3**](#)

[**Table A.4**](#)

[**Table A.5**](#)

[**Table A.6**](#)

[B A Brief Note on Excel](#)

[**Table B.1**](#)

List of Illustrations

[Chapter 1](#)

[**Figure 1.1** Results from a Gallup poll question ‘How well do you feel you understand the issue of global warming?’ that has been asked since 1989.](#)

[**Figure 1.2** A schematic describing the broad-scale subcomponents of the Earth system. Graphics reproduced with permission from the UK/NERC National Centre for Earth Observation. \(Image courtesy of NASA.\)](#)

[Chapter 2](#)

[**Figure 2.1**A schematic describing the scientific method.](#)

Figure 2.2 Example orders of magnitude found in geosciences. (Image courtesy of NASA.) The microscope photo of the soot particle is taken from Murr and Garza, *Atm. Env.*, **43**, 2683–2692, 2009. (Reproduced with permission of Elsevier.)

Figure 2.3 Examples of quantities expressed in scientific number format. (a) Earth from space (Image courtesy of NASA.) (b) Bankroll (Reproduced with permission of Andrew Magill.) (c) Human hair (Reproduced with permission of Bryan Bandli, Scanning Electron Microscopy Laboratory, University of Minnesota) (d) DNA Helix (Image courtesy of Richard Wheeler, http://commons.wikimedia.org/wiki/File:A-DNA,_B-DNA_and_Z-DNA.png).

Figure 2.4 A dataset that illustrates the potential pitfalls of blindly using the mean statistic. The mean value is denoted by the dashed horizontal line.

Figure 2.5 Summertime mean (June–August) temperatures (°C) at Lerwick (60.1°N, 1.2°W) for 1932–2010. Data are from the GISS Surface Temperature Analysis (GISTEMP) project.

Figure 2.6 Surface pressure chart for 1 August 2012. A deep low pressure system west of Ireland will carry a series of fronts across the UK. This system will remain near the UK until the end of the weekend. Plot courtesy of the UK Meteorological Office, Crown Copyright.

Chapter 3

Figure 3.1 (a) Construction of a golden rectangle; (b) the resulting golden spiral if the construction is repeated. Source: Wikimedia image courtesy of Dicklyon; (c,d) two examples of a golden spiral in the

natural world [(c) Snail shell image. Source: Wikimedia image courtesy of Chris 73; (d) Astral image. Source: Courtesy of the European Space Agency and NASA.]

Chapter 4

Figure 4.1 Anatomy of the $x - y$ linear graph.

Figure 4.2 Example linear plots with different values for the gradient a and the linear intercept b .

Figure 4.3 The general approach to graphing linear equations. The ratio $\Delta y/\Delta x$ defines the slope of the line.

Figure 4.4 Plots associated with example questions.

Figure 4.5 The buoyancy of an iceberg.

Chapter 5

Figure 5.1 Curve describing the exponential increase in population (billions) as a function of time (years).

Figure 5.2 Schematic explanation of exponential decay and half-life.

Figure 5.3 Plotting data from Gutenberg and Richter (1954).

Chapter 6

Figure 6.1 Two common depictions of accuracy versus precision. Top: target analogy to explain to the concept of precision and accuracy, where the true value is denoted by the centre or 'bull's-eye' of the target. Bottom: accuracy indicates proximity of measurement results to the true value, precision the repeatability or reproducibility of the measurement.

Figure 6.2 Descriptive error bars. Mean values (μ) with error bars for three experiments where experimental values (n) are taken 5, 50, and 500 times. The dots represent the individual data points. The true mean and standard deviation are given by the solid and horizontal lines, respectively. Note how the range of measured values increases with the number of individual measurements. The standard deviation (σ) includes about two thirds of the measurements. As you increase the number of measurements, the mean of the results will progressively get closer to the true mean.

Figure 6.3 A plot of a normal distribution. Each shaded band has a width of one standard deviation.

Figure 6.4 A schematic describing the relationship between the first, second, and third quartiles and standard deviations (σ) of a dataset. Figure from Wikimedia Commons. (Courtesy of Jhguch; retouched by Chen-Pan Liao.)

Figure 6.5 Examples of different graphs that can communicate your findings in a clear and concise manner.

Figure 6.6 Examples of different graphs that can communicate your findings in a clear and concise manner. The ozone dataset is for St Leonards, Edinburgh, and is supplied by AEA Energy and Environment.

Figure 6.7 Example scatterplots of two variables with the associated correlation coefficients given above each plot. This figure is adapted from Wikimedia Commons. (Courtesy of DenisBoigelot.)

Chapter 7

Figure 7.1 A right-angled triangle.

Figure 7.2 Special right-angled triangles: the 45° - 45° - 90° triangle (left) and the 30° - 60° - 90° triangle (right).

Figure 7.3 Mathematical terminology used to describe a circle. The relationship between degrees and radians (left) and between the radius and diameter (right).

Figure 7.4 Graphs of cosine and sine functions as a function of angle θ measured in degrees.

Figure 7.5 A non-right-angled triangle.

Figure 7.6 Triangles for Question 1.

Figure 7.7 The Colorado river. (Reproduced with permission of John Wiley and Sons.)

Figure 7.8 The angle at a survey site formed by two measurement sites.

Chapter 8

Figure 8.1 Resultant vectors.

Figure 8.2 Some example vectors.

Figure 8.3 Vector triangle.

Figure 8.4 Resolving vectors.

Figure 8.5 Triangle law for vector addition.

Figure 8.6 Airplane example.

Figure 8.7 Ocean buoy example.

Figure 8.8 Hawaiian Island Chain.

Chapter 9

Figure 9.1 A graphical interpretation of differentiation.

Figure 9.2 The slope of the secant line.

Figure 9.3 The functions $y = 5x^2 - 10x - 15$ and $y = x^3/3 - 5x^2/2 + 6x$.

Chapter 10

Figure 10.1 A graphical interpretation of integration.

Figure 10.2 The arbitrary continuous curve is approximated by a series of trapezoids between points a and b .

Chapter 11

A Answers to Problems

Figure A.1 Summertime mean (June–August) temperatures ($^{\circ}\text{C}$) at Geneva (46.2°N , 6.1°W), 1881–2010. Data is from the GISS Surface Temperature Analysis (GISTEMP) project. The dash (---) vertical line denotes the mean temperature value. Courtesy of NASA, GISS Surface Temperature Analysis (GISTEMP).

Figure A.2 Temperature of the Earth ($^{\circ}\text{C}$) as a function of depth (km). The symbols used are explained in the legend.

Figure A.3 Global population (billions of people) as a function of time (years). The solid black line is our linear model and the black squares are the estimates from United Nations, Department of Economic and Social Affairs, Population Division, *World Population Prospects: The 2010 Revision (2011)*. The dash (---) lines are the model estimates that account for a $\pm 25\%$ error in the population rate r .

Figure A.4 The radioactive decay of the rock specimen as a function of time.

Figure A.5 Gaussian distribution

$y = \frac{1}{\sqrt{2\pi}\sigma} \exp\{-(x - \mu)^2/2\sigma^2\}$ using $\mu = 50$, and $\sigma = 1, 5, 10, 25, 50$.

Figure A.6 Plot of temperature anomaly time series.

Figure A.7 Measuring the height of a tower.

Figure A.8 Measuring the height of a sand dune.

Figure A.9 The Great Pyramid at Giza.

Figure A.10 Sketch for forest fire problem.

Figure A.11 Dimensions of a larva pit.

Figure A.12 Graphical representation of Edinburgh wind data.

Figure A.13 River channel in plan view with flow measurements.

Figure A.14 Sketch for Question 4(b).

Figure A.15 Graph of $y = 2x^3 - 6x^2 - 18x$.

Figure A.16 Position, velocity and acceleration of ocean buoy.

Figure A.17 Displacement and velocity of seed pod.

Figure A.18 The form of the infection as a function of number of trees infected (arbitrary numbers).

Figure A.19 Photosynthetic uptake of carbon, 2009.

Figure A.20 Flying speed as a function of beetle mass (top) and log flying speed as a function of log beetle mass (bottom).

Figure A.21 Data and fitted model for Question 7.

Figure A.22 Annual deforestation rates (km^2/year) and the corresponding anomalies (km^2/year) as a

function of year. The dotted, dashed, and dot-dash lines denote the mean and the first and second standard deviation values about the mean, respectively.

Figure A.23 Linear models of Arctic sea ice extent.

Preface

This book includes the lectures and problem sets from the one-semester course 'Earth Modelling and Prediction' that I teach at the University of Edinburgh. The course is aimed at first-year geoscience undergraduates who want to understand the Earth and its evolving climate but do not have the necessary quantitative skills to move beyond qualitative studies. My primary and most ambitious objective for this course was to help students overcome the psychological barrier of *applying* mathematics to problems associated with the Earth. It is this barrier that artificially limits students' ability to gain a deeper understanding of the underlying science. My second objective was to show that the relatively simple mathematics covered in this course could be applied to learn something relevant to current areas of scientific research.

The focus of the book is the application of mathematics to scientifically relevant problems. Rather than being comprehensive, the material should be seen as providing a background for more advanced geoscience courses, which practise the application of mathematics and introduce the students to additional mathematics. I support the use of real data in teaching and so in recent years I have included progressively more exercises that involve the analysis of real measurements, many of which form the backdrop to a major news story in that year, for example, increased/decreased tropical deforestation rates or the reduction in the spatial extent of Arctic sea ice. I hope to include in future editions more varied data analysis problems that reflect the breadth of geoscience research.

I thank Patience Cowie, Roger Scrutton, and Roger Hipkin for recognizing the need for this course and for helping me to establish it at Edinburgh. For helping to teach topics within the course over the years I thank Patience Cowie, Godfrey Fitton, Gabriele Hegerl, Roger Hipkin, Ian Main, Chris Merchant, Mark Parrington, Simon Tett, and Thorvaldur Thordarson. I thank all the tutors who helped to make the course work well: Amber Annett, Louise Barron, Dave Bell, Anthony Bloom, Matthew Brolly, Iain Cameron, Ruth Carley, Craig Duguid, Leon Kapetas, Simon King, Jack Lonsdale, Malcolm McMillan, Simone Morak, Heather Nicolson, Katie Noak, Luke Ridley, Robert Shore, Luke Smallman, Lorna Street, Oliver Sus, Sarah Touati, Matthew Unterman, Lucia Viegas, and Adam Wilson. Finally, I thank Martin Wooster (King's College London) for proofreading and providing useful comments on an earlier draft of the manuscript.

Paul I. Palmer

University of Edinburgh

May 2013

1

How Do You Know that Global Warming Is Not a Hoax?

The title of this introductory chapter is the question I pose at the start of my course in Edinburgh. It seems like a ridiculous question to ask a bunch of bright young students, especially ones who have chosen to study the Earth system. But up until walking through the doors of the university many students have not had the resources, inclination, and/or ability to *question* what they are told; the key to being an effective scientist is to ask the right questions, ones that probe at the very heart of the problem being studied. I provide the student with four possible choices to answer the question and ask for a show of hands:

1. popular media (internet, TV, radio, newspapers);
2. rigorous scientific reasoning and/or debate;
3. (blind) faith in scientists; or
4. other.

Typically, choice 1 represents the vast majority of hands. Why? Because we are bombarded with scientific and political coverage of climate change. Why is this dangerous? Because companies need to sell newspapers and to get people to watch TV, and politicians are invariably biased in their opinions. Much of the coverage is accurate but some programmes are biased, loosely based on fact, with a damaging effect on the science education of the general public. Sensationalism about Earth's climate (particularly looking to the future) is rife, but some aspects

of Earth's climate are *genuinely* remarkable and awe-inspiring. So how do you know what to believe?

Choice 2 often represents the second highest show of hands, but a much smaller proportion than choice 1. This is fine up to a point. Scientists are some of the biggest sceptics around and are generally very careful about what they say. For instance, we see later in this chapter that the wording used in the Intergovernmental Panel on Climate Change (IPCC) report¹ has very strict statistical interpretation that is difficult to misinterpret. But you only learn from the scientists what they tell you. How did they reach their conclusions? Could they have approached the problem from a different perspective and reached a different conclusion? With the renewed call for transparency in science, particularly related to climate, most data used to draw conclusions about Earth's climate are online and freely available to download. Often the only barrier to pursuing option 2, given that data are now freely available, is the confidence to understand and interrogate quantitative data. The aim of this book is to increase that confidence.

This mix of responses is reasonably similar to the general public response to the question 'How well do you feel you understand the issue of global warming?' that has been asked frequently by Gallup (www.gallup.com) for the past quarter century ([Figure 1.1](#)). For this admittedly crude comparison I have equated 'Great deal' with 'Rigorous scientific reasoning', 'Fair amount' with 'Popular media', and 'Only a little' with '(Blind) faith in scientists'.

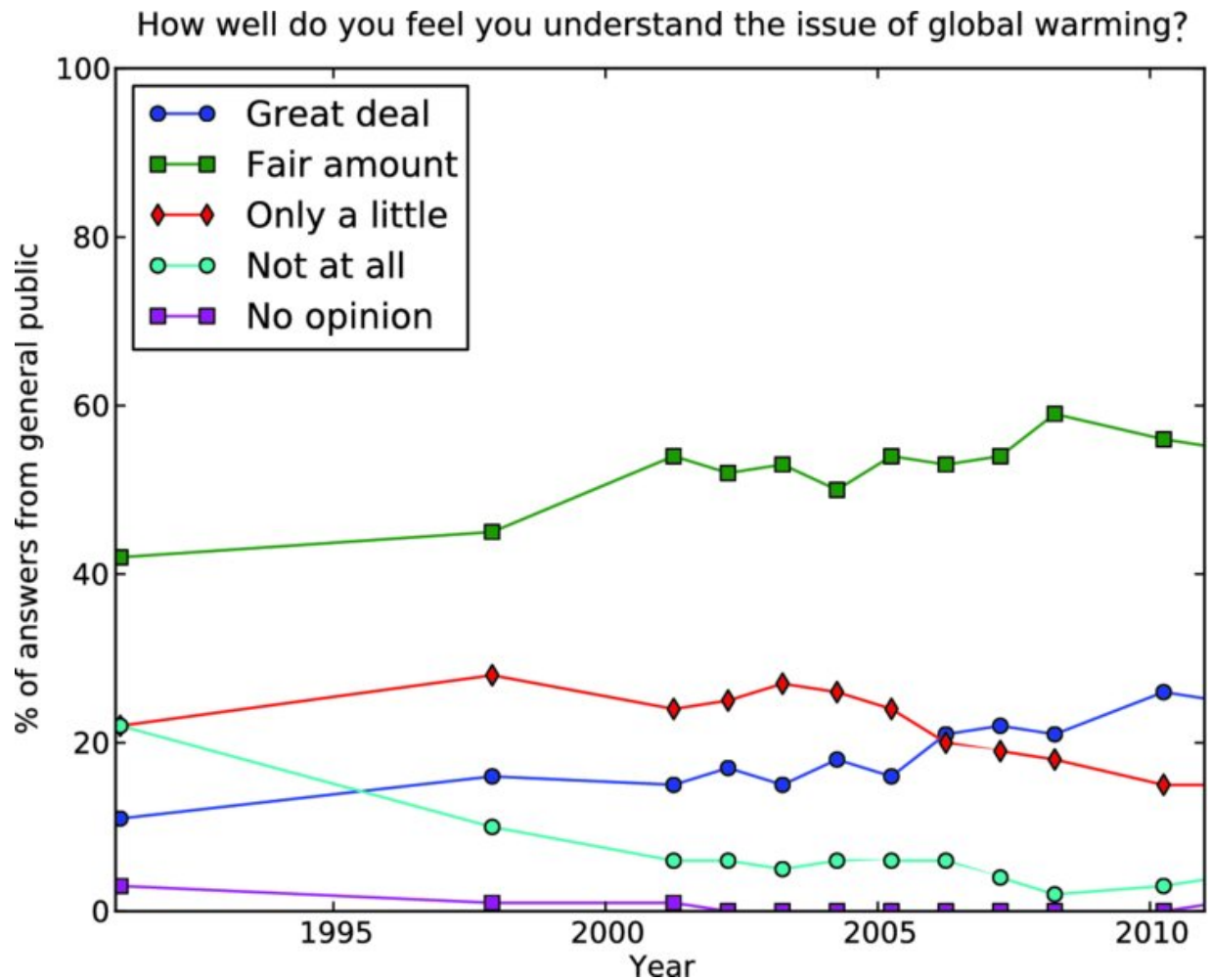


Figure 1.1 Results from a Gallup poll question ‘How well do you feel you understand the issue of global warming?’ that has been asked since 1989.

How can mathematics help? In simple terms, mathematics (at this level) is a tool that allows us to move far beyond what we can learn from descriptive analysis. How much has sea ice changed? If we use the current rate of change, how long will it be before the Arctic is free of ice? These are simple example questions that cannot be answered without mathematics.

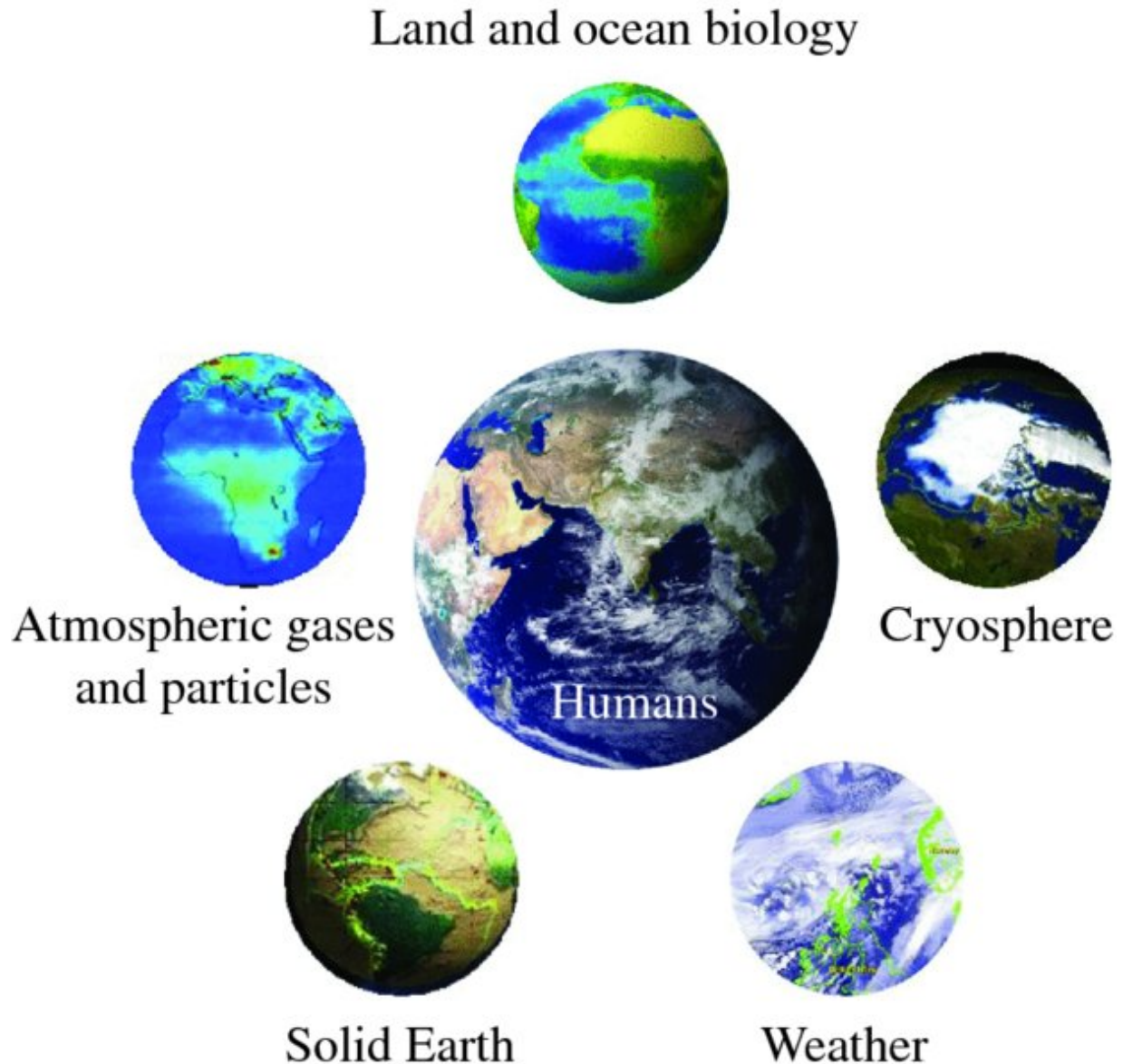


Figure 1.2 A schematic describing the broad-scale subcomponents of the Earth system. Graphics reproduced with permission from the UK/NERC National Centre for Earth Observation. (Image courtesy of NASA.)

The Earth system: how do we know what we know?

I define the Earth system as the land, ocean, and atmosphere, all the physical, chemical, biological, and

social processes and their interactions ([Figure 1.2](#)). This is a big unwieldy interconnected system that is coupled on a wide spectrum of spatial and temporal scales. To minimize the risk of discussing current science results that might be superseded by new data, I have decided to focus on *how* scientists generally know what they know about the Earth system and the recent role of human activity and not *what* they know:

- First, we have a basic physical understanding of the Earth. We know, for example, about the heat-trapping properties of gases in the atmosphere, based on work first started in the nineteenth century. Another example is continental drift, a theory describing how Earth's continents move relative to each other, which has been known since the twentieth century. These are well-established science theories that have stood up to decades/centuries of scientific scrutiny.
- Second, we have circumstantial evidence. We make qualitative connections between observations of disparate quantities and results from computer models² of the Earth system, for example, warming of oceans, lands, and the lower atmosphere, cooling of the middle atmosphere, and increases in water vapour.
- Third, we have palaeoclimate evidence. We can reconstruct past climate using a variety of data, for example, ice core, lake sediment core, coral reefs, pollen. This places contemporary warming trends in the longer-term context. Although there is debate about whether the past is any guide to the future, they do provide us a history of how Earth has behaved in the past.
- Finally, we have so-called 'fingerprint' evidence. The underlying philosophy is that individual (natural and

human-driven) processes will leave their own unique signature (or fingerprint) on measurements of the Earth. By comparing these data that naturally include these signatures with computer models of climate with/without descriptions of the processes responsible for these signatures we can understand the importance of individual processes. This can also potentially identify the need for additional processes that are currently not present in the model.

It is important to acknowledge that several independent lines of inquiry are used to investigate phenomena and provide evidence to test a hypothesis. The IPCC is testing the overarching hypothesis that human activity has determined recent changes in climate. As we will see in the next chapter, the hypothesis is right at the crux of the *scientific method*. In successive IPCC reports the headline result has been stronger and stronger:

- **1995:** The balance of evidence *suggests* a discernable human influence on global climate.
- **2001:** Most of the observed warming over the last 50 years is *likely to have been due* to the increase in greenhouse gas concentrations.
- **2007:** Most of the observed increase in globally averaged temperatures since the mid-twentieth century is *very likely due* to the observed increase in anthropogenic greenhouse gas concentrations.

In the IPCC nomenclature the term ‘likely’ refers to a probability greater than 66% and ‘very likely’ to a probability greater than 90%. In 2001 the IPCC was more than 66% certain that climate change was caused by human activity. By 2007 it was more than 90% certain that recent climate change is due to anthropogenic greenhouse

gas concentrations. And most recently, in 2013, the IPCC increased this confidence to 95%. It is possible that climate change is due to other causes, but the IPCC regards this as unlikely. It is unfortunate that this level of scientific 'honesty' also represents an inroad to climate scepticism.

Notes

- ¹ A report prepared by a subset of leading climate scientists that summarizes the state of the science. The latest report can be found at www.ipcc.ch
- ² A model in this instance is a collection of interrelated equations, written in a computer language, that describe, for example, the physics, chemistry, and biology of the atmosphere and ocean. Without a computer, evaluating these equations would be an intractable task. In fact some of the fastest computers in the world are dedicated to studying Earth's climate.

2

Preamble:

This chapter lays out many core mathematical skills that are important but do not fit neatly into other chapters.

2.1 The scientific method: pushing back the frontiers of ignorance

We start by introducing the idea of the scientific method, which describes a general series of steps for investigating phenomena. You will already be familiar with many of the steps but it is useful to go over the basics. [Figure 2.1](#) illustrates the basic steps of the scientific method (see also boxed text below).

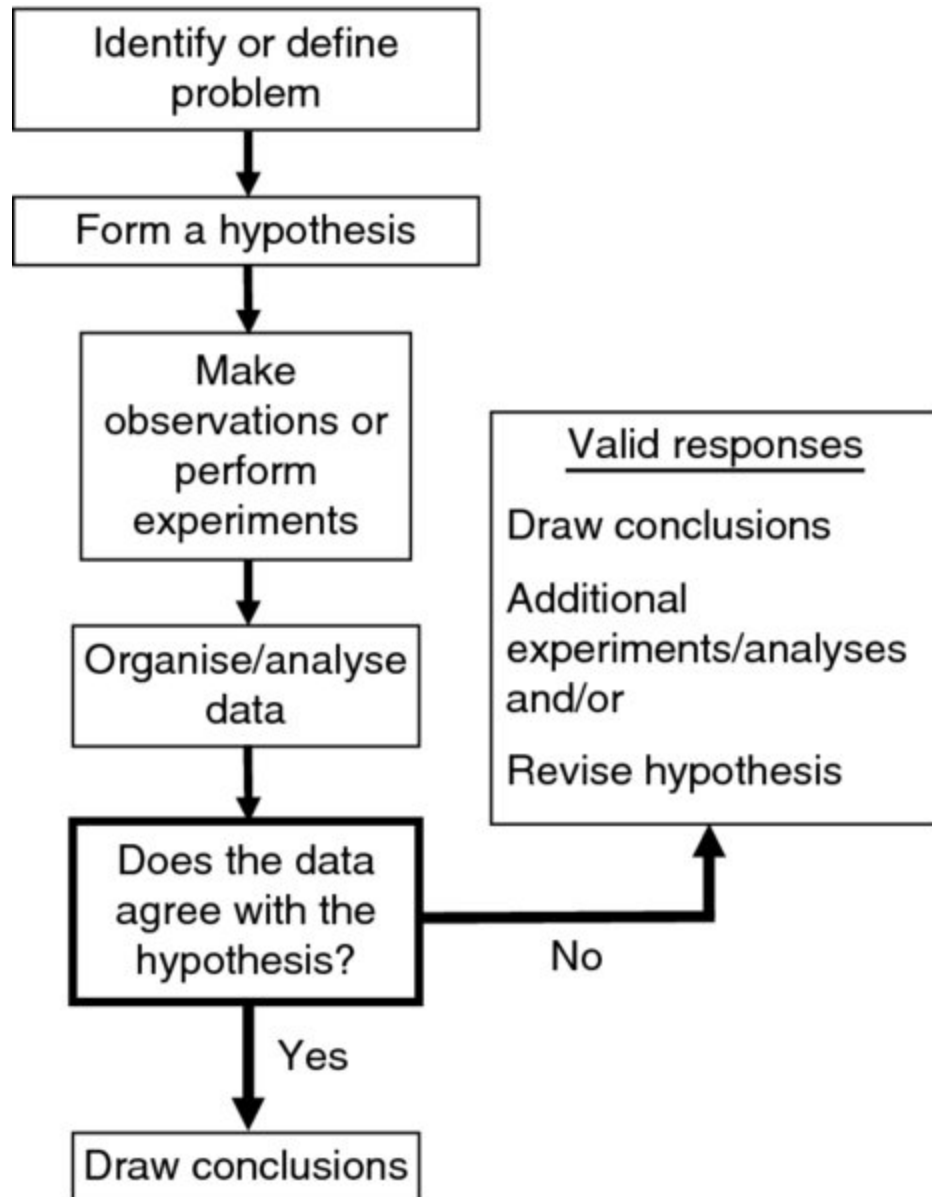


Figure 2.1 A schematic describing the scientific method.

We start by identifying or defining a problem to investigate. It might be that a 'problem' can be split up into a number of sub-problems.

We follow this by forming a hypothesis, an idea of what/how we expect the problem to be once it is measured. The hypothesis can be as simple as the expected value of a measurement or as complicated as how an object will respond to a change in its environment. It is important to

note that the hypothesis must be formed prior to the measurement, otherwise it compromises the validity of the conclusion we might draw from the method.

We make some observations or we perform an experiment to test the hypothesis. We aim to improve knowledge of the system by measurement. For many problems, data may already be available, in which case we move on to the next step.

We organize or analyse the data. 'Organizing the data might involve gathering together or combining different sets of data. Data analysis describes a whole range of techniques, some of which we will discuss in later chapters. In both this step and the last, we must pay careful attention to measurement error, otherwise our analysis may result in erroneous conclusions. We discuss errors in Chapter 6.

Finally, we self-reflect on our experiment. Do the data agree with the hypothesis? Is the answer definitive? Are other explanations possible? This is an important step in the overall scientific method (therefore marked in bold) and is what distinguishes the method from less rigorous pseudo-science methods. Depending on the nature of your experiment, you may have learnt something if the data agree or disagree with the original hypothesis. So you might choose to draw reasonable conclusions at this point. If the data do not agree with the original hypothesis, other valid responses might include additional analysis or additional experiments to refine the original hypothesis. Or you may choose to completely revise the hypothesis and go through the whole process again.

Because of the importance of this method we will return to many of these key concepts, particularly the self-reflection, throughout the book.