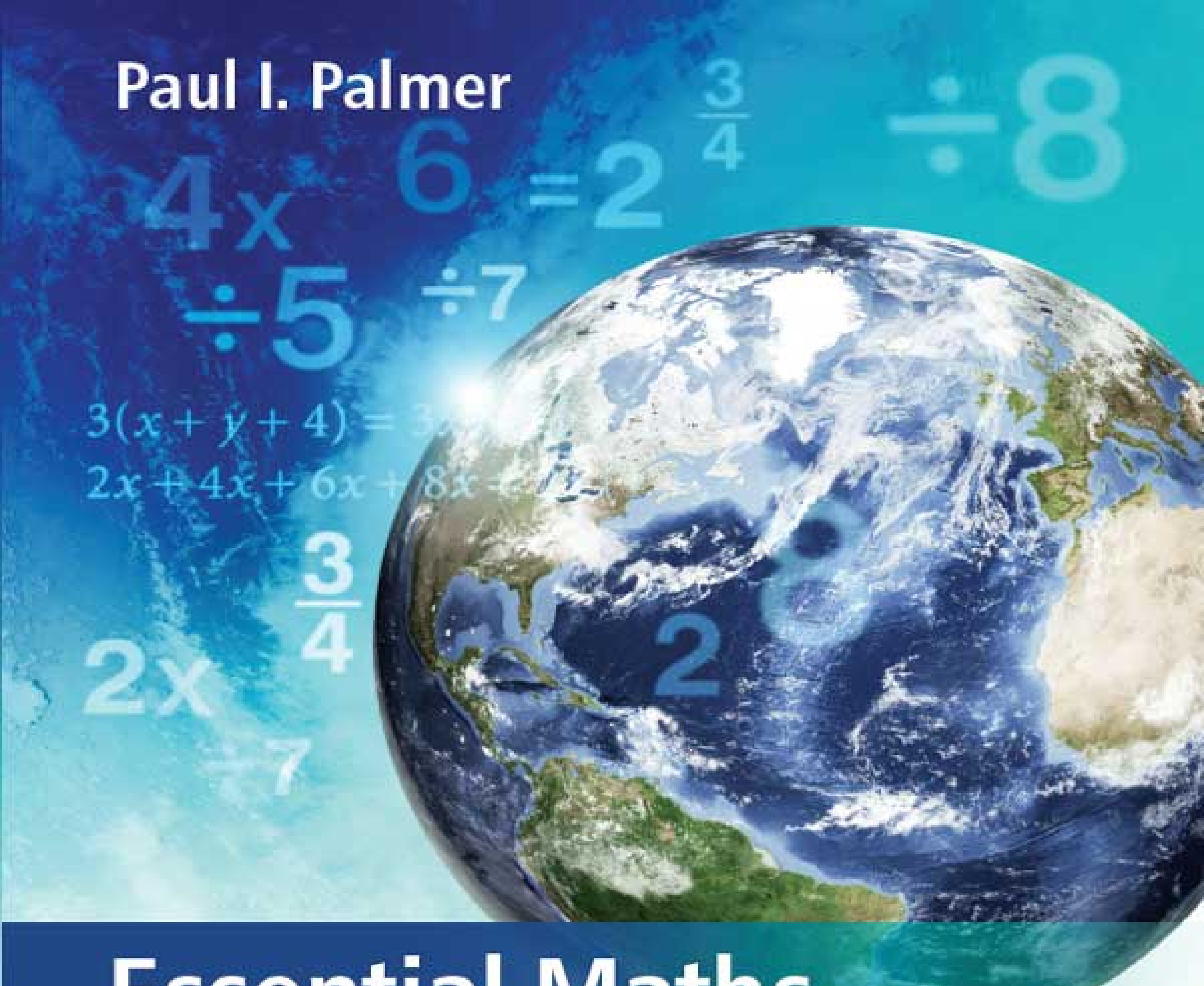
Paul I. Palmer



Essential Maths for Geoscientists

An Introduction

with website

WILEY Blackwell

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An Introduction

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To A.L.M., L.M.P., and J.S.P.

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function of year. The dotted, dashed, and dot-dash lines denote the mean and the first and second standard deviation values about the mean, respectively.

Figure A.23 Linear models of Arctic sea ice extent.

Preface

This book includes the lectures and problem sets from the one-semester course ‘Earth Modelling and Prediction’ that I teach at the University of Edinburgh. The course is aimed at first-year geoscience undergraduates who want to understand the Earth and its evolving climate but do not have the necessary quantitative skills to move beyond qualitative studies. My primary and most ambitious objective for this course was to help students overcome the psychological barrier of *applying* mathematics to problems associated with the Earth. It is this barrier that artificially limits students’ ability to gain a deeper understanding of the underlying science. My second objective was to show that the relatively simple mathematics covered in this course could be applied to learn something relevant to current areas of scientific research.

The focus of the book is the application of mathematics to scientifically relevant problems. Rather than being comprehensive, the material should be seen as providing a background for more advanced geoscience courses, which practise the application of mathematics and introduce the students to additional mathematics. I support the use of real data in teaching and so in recent years I have included progressively more exercises that involve the analysis of real measurements, many of which form the backdrop to a major news story in that year, for example, increased/decreased tropical deforestation rates or the reduction in the spatial extent of Arctic sea ice. I hope to include in future editions more varied data analysis problems that reflect the breadth of geoscience research.

I thank Patience Cowie, Roger Scrutton, and Roger Hipkin for recognizing the need for this course and for helping me to establish it at Edinburgh. For helping to teach topics within the course over the years I thank Patience Cowie, Godfrey Fitton, Gabriele Hegerl, Roger Hipkin, Ian Main, Chris Merchant, Mark Parrington, Simon Tett, and Thorvaldur Thordarson. I thank all the tutors who helped to make the course work well: Amber Annett, Louise Barron, Dave Bell, Anthony Bloom, Matthew Brolly, Iain Cameron, Ruth Carley, Craig Duguid, Leon Kapetas, Simon King, Jack Lonsdale, Malcolm McMillan, Simone Morak, Heather Nicolson, Katie Noak, Luke Ridley, Robert Shore, Luke Smallman, Lorna Street, Oliver Sus, Sarah Touati, Matthew Unterman, Lucia Viegas, and Adam Wilson. Finally, I thank Martin Wooster (King's College London) for proofreading and providing useful comments on an earlier draft of the manuscript.

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University of Edinburgh
May 2013

1
How Do You Know that Global Warming Is Not a Hoax?

The title of this introductory chapter is the question I pose at the start of my course in Edinburgh. It seems like a ridiculous question to ask a bunch of bright young students, especially ones who have chosen to study the Earth system. But up until walking through the doors of the university many students have not had the resources, inclination, and/or ability to *question* what they are told; the key to being an effective scientist is to ask the right questions, ones that probe at the very heart of the problem being studied. I provide the student with four possible choices to answer the question and ask for a show of hands:

1. popular media (internet, TV, radio, newspapers);
2. rigorous scientific reasoning and/or debate;
3. (blind) faith in scientists; or
4. other.

Typically, choice 1 represents the vast majority of hands. Why? Because we are bombarded with scientific and political coverage of climate change. Why is this dangerous? Because companies need to sell newspapers and to get people to watch TV, and politicians are invariably biased in their opinions. Much of the coverage is accurate but some programmes are biased, loosely based on fact, with a damaging effect on the science education of the general public. Sensationalism about Earth's climate (particularly looking to the future) is rife, but some aspects of Earth's climate are *genuinely* remarkable and awe-inspiring. So how do you know what to believe?

Choice 2 often represents the second highest show of hands, but a much smaller proportion than choice 1. This is fine up to a point. Scientists are some of the biggest sceptics around and are generally very careful about what they say. For instance, we see later in this chapter that the wording used in the Intergovernmental Panel on Climate Change (IPCC) report[1] has very strict statistical interpretation that is difficult to misinterpret. But you only learn from the scientists what they tell you. How did they reach their conclusions? Could they have approached the problem from a different perspective and reached a different conclusion? With the renewed call for transparency in science, particularly related to climate, most data used to draw conclusions about Earth's climate are online and freely available to download. Often the only barrier to pursuing option 2, given that data are now freely available, is the confidence to understand and interrogate quantitative data. The aim of this book is to increase that confidence.

This mix of responses is reasonably similar to the general public response to the question 'How well do you feel you understand the issue of global warming?' that has been asked frequently by Gallup (www.gallup.com) for the past quarter century (Figure 1.1). For this admittedly crude comparison I have equated 'Great deal' with 'Rigorous scientific reasoning', 'Fair amount' with 'Popular media', and 'Only a little' with '(Blind) faith in scientists'.

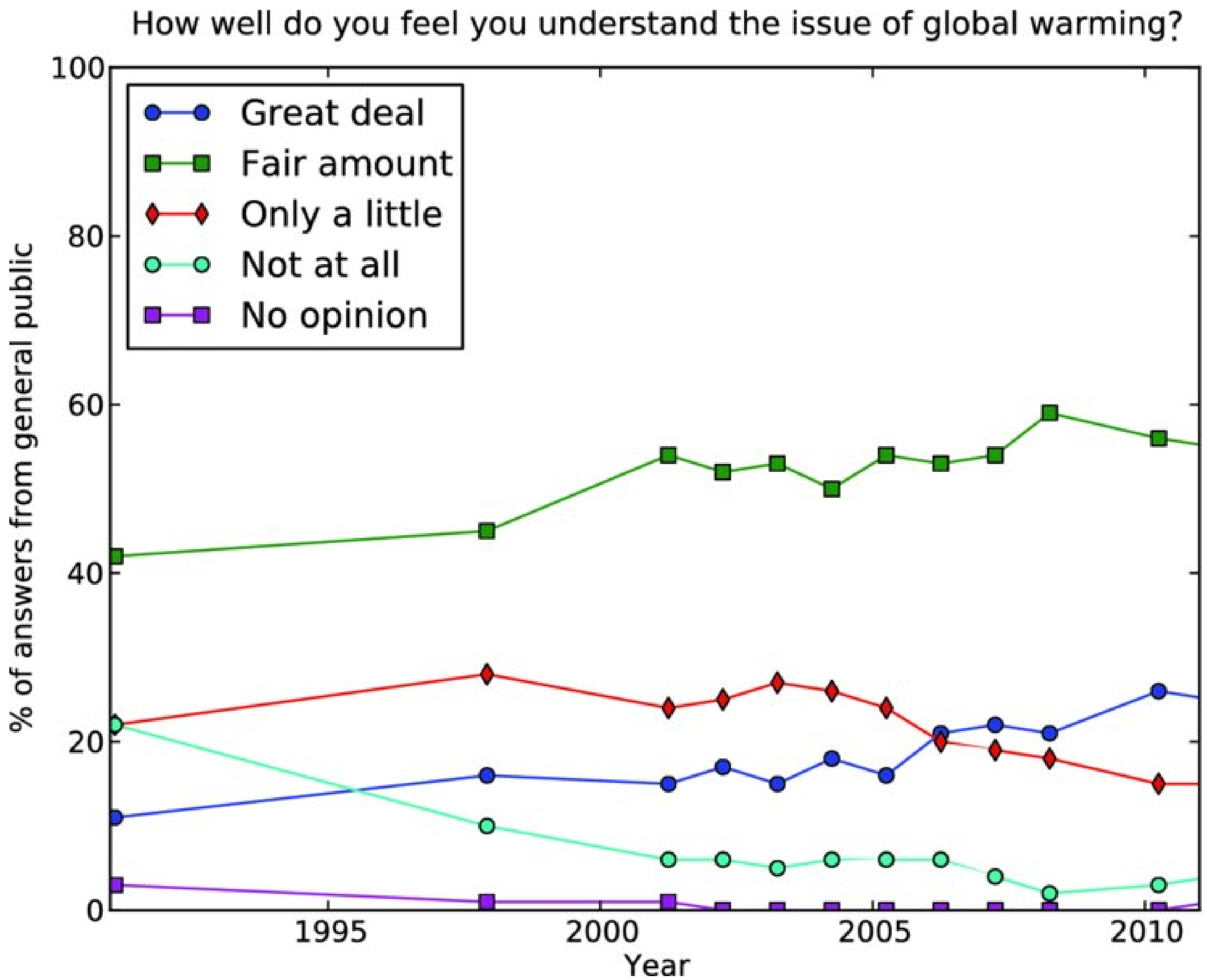

Figure 1.1 Results from a Gallup poll question 'How well do you feel you understand the issue of global warming?' that has been asked since 1989.

How can mathematics help? In simple terms, mathematics (at this level) is a tool that allows us to move far beyond what we can learn from descriptive analysis. How much has sea ice changed? If we use the current rate of change, how long will it be before the Arctic is free of ice? These are simple example questions that cannot be answered without mathematics.

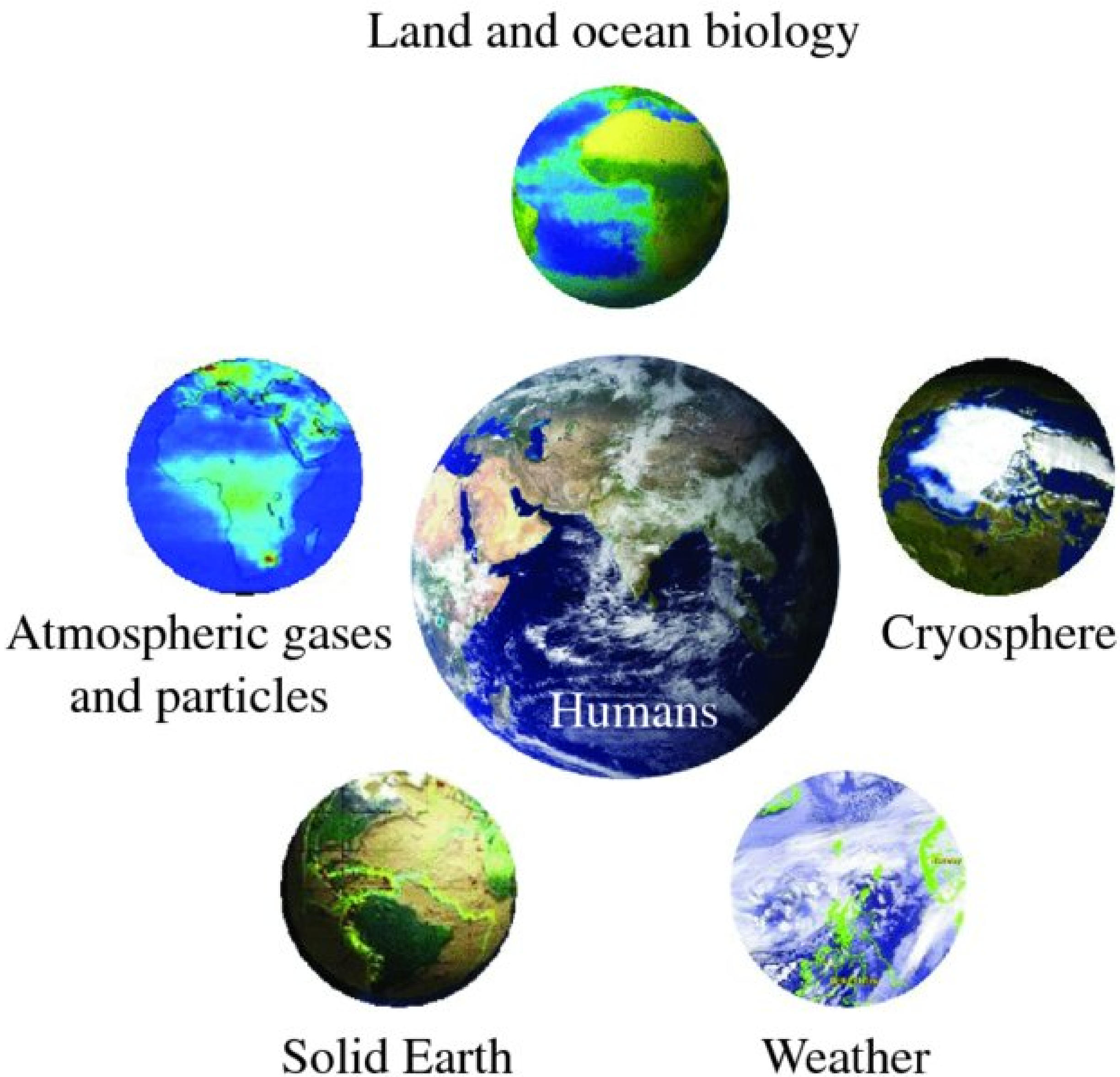

Figure 1.2 A schematic describing the broad-scale subcomponents of the Earth system. Graphics reproduced with permission from the UK/NERC National Centre for Earth Observation. (Image courtesy of NASA.)

The Earth system: how do we know what we know?

I define the Earth system as the land, ocean, and atmosphere, all the physical, chemical, biological, and social processes and their interactions (Figure 1.2). This is a big unwieldy interconnected system that is coupled on a wide spectrum of spatial and temporal scales. To minimize the risk of discussing current science results that might be superseded by new data, I have decided to focus on *how* scientists generally know what they know about the Earth system and the recent role of human activity and not *what* they know:

- First, we have a basic physical understanding of the Earth. We know, for example, about the heat-trapping properties of gases in the atmosphere, based on work first started in the nineteenth century. Another example is continental drift, a theory describing how Earth's continents move relative to each other, which has been known since the twentieth century. These are well-established science theories that have stood up to decades/centuries of scientific scrutiny.
- Second, we have circumstantial evidence. We make qualitative connections between observations of disparate quantities and results from computer models[2] of the Earth system, for example, warming of oceans, lands, and the lower atmosphere, cooling of the middle atmosphere, and increases in water vapour.
- Third, we have palaeoclimate evidence. We can reconstruct past climate using a variety of data, for example, ice core, lake sediment core, coral reefs, pollen. This places contemporary warming trends in the longer-term context. Although there is debate about whether the past is any guide to the future, they do provide us a history of how Earth has behaved in the past.
- Finally, we have so-called 'fingerprint' evidence. The underlying philosophy is that individual (natural and

human-driven) processes will leave their own unique signature (or fingerprint) on measurements of the Earth. By comparing these data that naturally include these signatures with computer models of climate with/without descriptions of the processes responsible for these signatures we can understand the importance of individual processes. This can also potentially identify the need for additional processes that are currently not present in the model.

It is important to acknowledge that several independent lines of inquiry are used to investigate phenomena and provide evidence to test a hypothesis. The IPCC is testing the overarching hypothesis that human activity has determined recent changes in climate. As we will see in the next chapter, the hypothesis is right at the crux of the *scientific method*. In successive IPCC reports the headline result has been stronger and stronger:

- **1995:** The balance of evidence *suggests* a discernable human influence on global climate.
- **2001:** Most of the observed warming over the last 50 years is *likely to have been due* to the increase in greenhouse gas concentrations.
- **2007:** Most of the observed increase in globally averaged temperatures since the mid-twentieth century is *very likely due* to the observed increase in anthropogenic greenhouse gas concentrations.

In the IPCC nomenclature the term ‘likely’ refers to a probability greater than 66% and ‘very likely’ to a probability greater than 90%. In 2001 the IPCC was more than 66% certain that climate change was caused by human activity. By 2007 it was more than 90% certain that recent climate change is due to anthropogenic greenhouse gas concentrations. And most recently, in 2013, the IPCC increased this confidence to 95%. It is possible that climate change is due to other causes, but the IPCC regards this as unlikely. It is unfortunate that this level of scientific 'honesty' also represents an inroad to climate scepticism.

Notes

[1] A report prepared by a subset of leading climate scientists that summarizes the state of the science. The latest report can be found at www.ipcc.ch

[2] A model in this instance is a collection of interrelated equations, written in a computer language, that describe, for example, the physics, chemistry, and biology of the atmosphere and ocean. Without a computer, evaluating these equations would be an intractable task. In fact some of the fastest computers in the world are dedicated to studying Earth's climate.

2
Preamble:

This chapter lays out many core mathematical skills that are important but do not fit neatly into other chapters.

2.1 The scientific method: pushing back the frontiers of ignorance

We start by introducing the idea of the scientific method, which describes a general series of steps for investigating phenomena. You will already be familiar with many of the steps but it is useful to go over the basics. Figure 2.1 illustrates the basic steps of the scientific method (see also boxed text below).

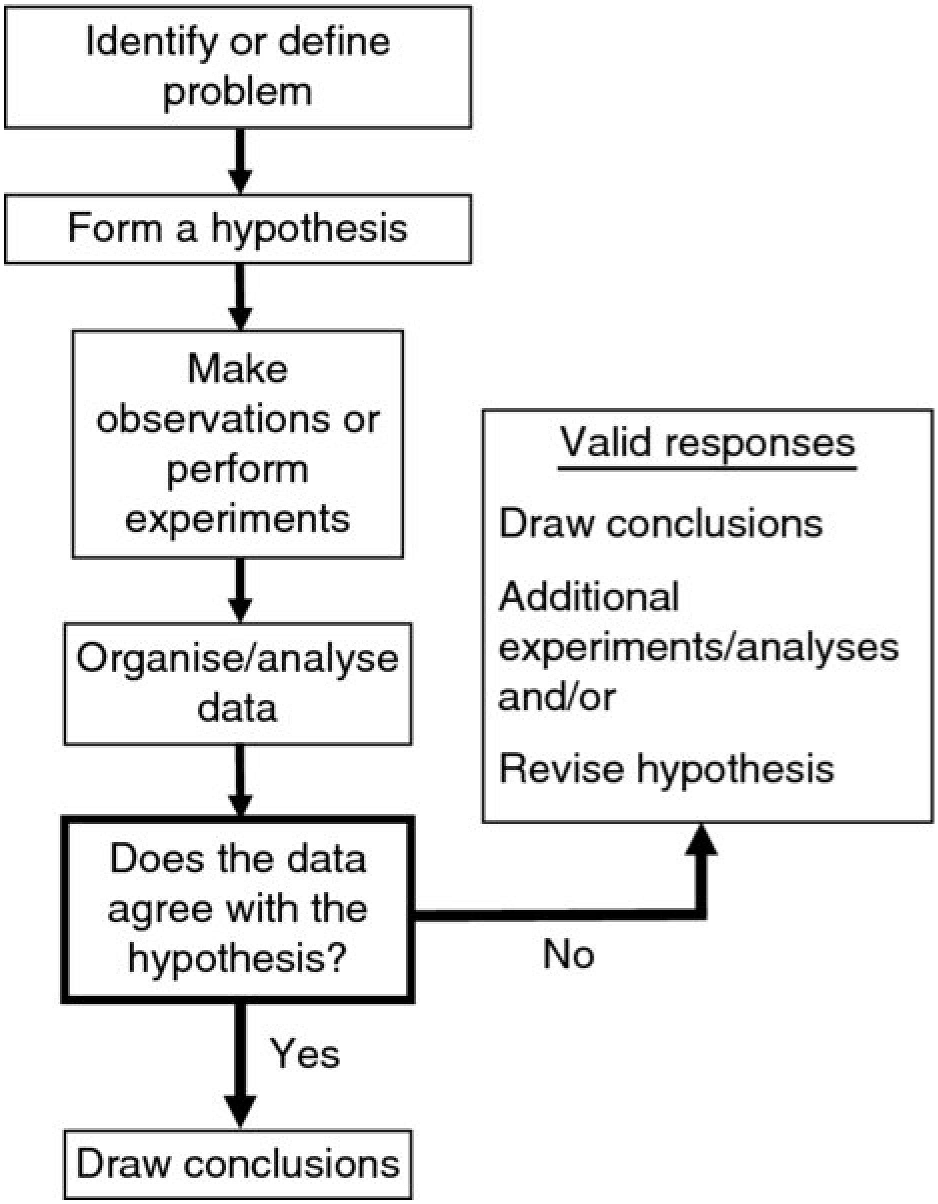

Figure 2.1 A schematic describing the scientific method.

We start by identifying or defining a problem to investigate. It might be that a 'problem' can be split up into a number of sub-problems.

We follow this by forming a hypothesis, an idea of what/how we expect the problem to be once it is measured. The hypothesis can be as simple as the expected value of a measurement or as complicated as how an object will respond to a change in its environment. It is important to note that the hypothesis must be formed prior to the measurement, otherwise it compromises the validity of the conclusion we might draw from the method.

We make some observations or we perform an experiment to test the hypothesis. We aim to improve knowledge of the system by measurement. For many problems, data may already be available, in which case we move on to the next step.

We organize or analyse the data. ‘Organizing the data might involve gathering together or combining different sets of data. Data analysis describes a whole range of techniques, some of which we will discuss in later chapters. In both this step and the last, we must pay careful attention to measurement error, otherwise our analysis may result in erroneous conclusions. We discuss errors in Chapter 6.

Finally, we self-reflect on our experiment. Do the data agree with the hypothesis? Is the answer definitive? Are other explanations possible? This is an important step in the overall scientific method (therefore marked in bold) and is what distinguishes the method from less rigorous pseudo-science methods. Depending on the nature of your experiment, you may have learnt something if the data agree or disagree with the original hypothesis. So you might choose to draw reasonable conclusions at this point. If the data do not agree with the original hypothesis, other valid responses might include additional analysis or additional experiments to refine the original hypothesis. Or you may choose to completely revise the hypothesis and go through the whole process again.

Because of the importance of this method we will return to many of these key concepts, particularly the self-reflection, throughout the book.