

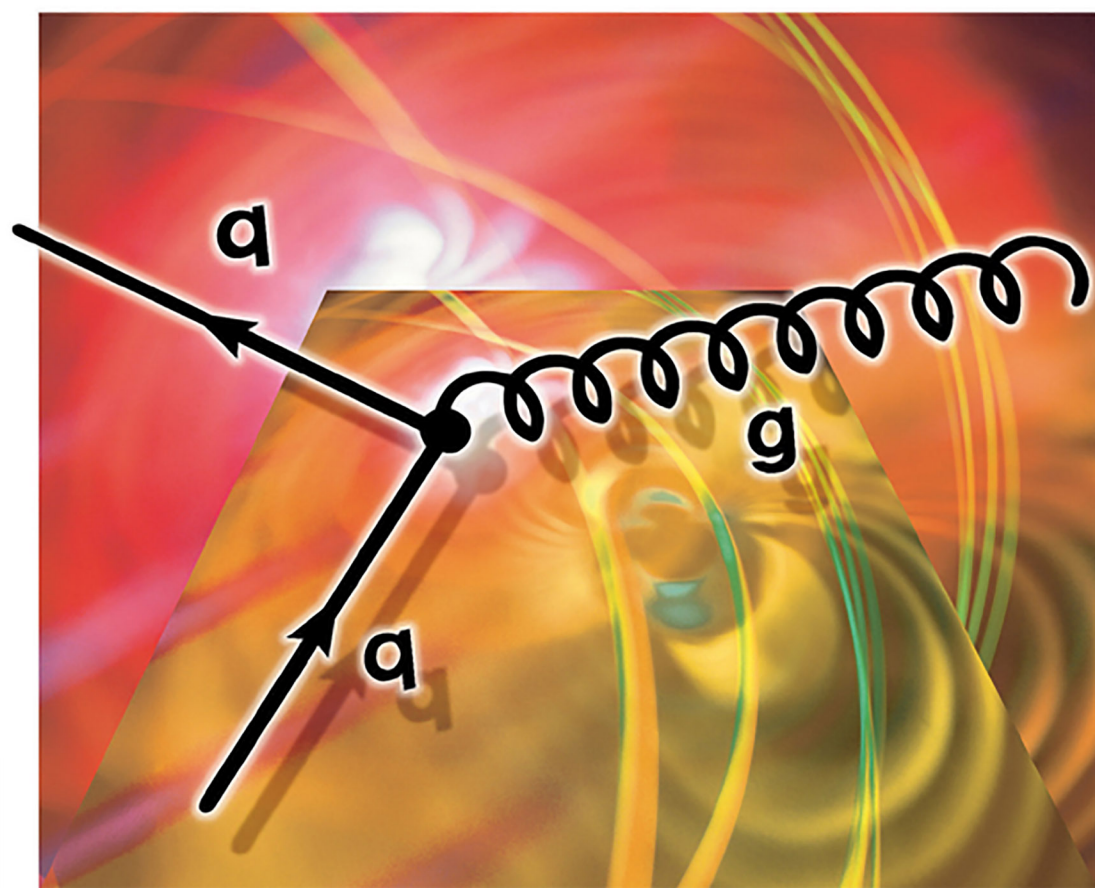
PHYSICS TEXTBOOK

David Griffiths

WILEY-VCH

# Introduction to Elementary Particles

Second, Revised Edition





*David Griffiths*

**Introduction to Elementary  
Particles**





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Second, Revised Edition



**WILEY-  
VCH**

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## Preface to the First Edition

This introduction to the theory of elementary particles is intended primarily for advanced undergraduates who are majoring in physics. Most of my colleagues consider this subject inappropriate for such an audience – mathematically too sophisticated, phenomenologically too cluttered, insecure in its foundations, and uncertain in its future. Ten years ago I would have agreed. But in the last decade the dust has settled to an astonishing degree, and it is fair to say that elementary particle physics has come of age. Although we obviously have much more to learn, there now exists a coherent and unified theoretical structure that is simply too exciting and important to save for graduate school or to serve up in diluted qualitative form as a subunit of modern physics. I believe the time has come to integrate elementary particle physics into the standard undergraduate curriculum.

Unfortunately, the research literature in this field is clearly inaccessible to undergraduates, and although there are now several excellent graduate texts, these call for a strong preparation in advanced quantum mechanics, if not quantum field theory. At the other extreme, there are many fine popular books and a number of outstanding *Scientific American* articles. But very little has been written specifically for the undergraduate. This book is an effort to fill that need. It grew out of a one-semester elementary particles course I have taught from time to time at Reed College. The students typically had under their belts a semester of electromagnetism (at the level of Lorrain and Corson), a semester of quantum mechanics (at the level of Park), and a fairly strong background in special relativity.

In addition to its principal audience, I hope this book will be of use to beginning graduate students, either as a primary text, or as preparation for a more sophisticated treatment. With this in mind, and in the interest of greater completeness and flexibility, I have included more material here than one can comfortably cover in a single semester. (In my own courses I ask the students to read Chapters 1 and 2 on their own, and begin the lectures with Chapter 3. I skip Chapter 5 altogether, concentrate on Chapters 6 and 7, discuss the first two sections of Chapter 8, and then jump to Chapter 10.) To assist the reader (and the teacher) I begin each chapter with a brief indication of its purpose and content, its prerequisites, and its role in what follows.

This book was written while I was on sabbatical at the Stanford Linear Accelerator Center, and I would like to thank Professor Sidney Drell and the other members of the Theory Group for their hospitality.

DAVID GRIFFITHS  
1986



## Preface to the Second Edition

It is 20 years since the first edition of this book was published, and it is both gratifying and distressing to reflect that it remains, for the most part, reasonably up-to-date. There are, to be sure, some gross lacunae – the existence of the top quark had not been confirmed back then, and neutrinos were generally assumed (for no very good reason) to be massless. But the Standard Model, which is, in essence, the subject of the book, has proved to be astonishingly robust. This is a tribute to the theory, and at the same time an indictment of our collective imagination. I don't think there has been a comparable period in the history of elementary particle physics in which so little of a truly revolutionary nature has occurred. What about neutrino oscillations? Indeed: a fantastic story (I have added a chapter on the subject); and yet, this extraordinary phenomenon fits so comfortably into the Standard Model that one might almost say, in retrospect (of course), that it would have been more surprising if it had *not* been so. How about supersymmetry and string theory? Yes, but these must for the moment be regarded as speculations (I have added a chapter on contemporary theoretical developments). As far as solid experimental confirmation goes, the Standard Model (with neutrino masses and mixing) still rules.

In addition to the two new chapters already mentioned, I have brought the history up-to-date in Chapter 1, shortened Chapter 5, provided (I hope) a more compelling introduction to the Golden Rule in Chapter 6, and eliminated most of the old Chapter 8 on electromagnetic form factors and scaling (this was crucially important in interpreting the deep inelastic scattering experiments that put the quark model on a secure footing, but no one today doubts the existence of quarks, and the technical details are no longer so essential). What remains of Chapter 8 is now combined with the old Chapter 9 to make a new chapter on hadrons. Finally, I have prepared a complete solution manual (available free from the publisher, though only – I regret – to course instructors). Beyond this the changes are relatively minor.

Many people have sent me suggestions and corrections, or patiently answered my questions. I cannot thank everyone, but I would like to acknowledge some of those who were especially helpful: Guy Blaylock (UMass), John Boersma (Rochester), Carola Chinellato (Brazil), Eugene Commins (Berkeley), Mimi Gerstell (Caltech),

Nahmin Horwitz (Syracuse), Richard Kass (Ohio State), Janis McKenna (UBC), Jim Napolitano (RPI), Nic Nigro (Seattle), John Norbury (UW-Milwaukee), Jason Quinn (Notre Dame), Aaron Roodman (SLAC), Natthi Sharma (Eastern Michigan), Steve Wasserbeach (Haverford), and above all Pat Burchat (Stanford).

Part of this work was carried out while I was on sabbatical, at Stanford and SLAC, and I especially thank Patricia Burchat and Michael Peskin for making this possible.

DAVID GRIFFITHS  
2008

# Formulas and Constants

## Particle Data

Mass in  $\text{MeV}/c^2$ , lifetime in seconds, charge in units of the proton charge.

### Leptons (spin 1/2)

Generation	Flavor	Charge	Mass*	Lifetime	Principal Decays
first	$e$ (electron)	$-1$	0.510999	$\infty$	$-$
	$\nu_e$ ( $e$ neutrino)	0	0	$\infty$	$-$
second	$\mu$ (muon)	$-1$	105.659	$2.19703 \times 10^{-6}$	$e\nu_\mu\bar{\nu}_e$
	$\nu_\mu$ ( $\mu$ neutrino)	0	0	$\infty$	$-$
third	$\tau$ (tau)	$-1$	1776.99	$2.91 \times 10^{-13}$	$e\nu_\tau\bar{\nu}_e, \mu\nu_\tau\bar{\nu}_\mu, \pi^-\nu_\tau$
	$\nu_\tau$ ( $\tau$ neutrino)	0	0	$\infty$	$-$

\*Neutrino masses are extremely small, and for most purposes can be taken to be zero; for details see Chapter 11.

### Quarks (spin 1/2)

Generation	Flavor	Charge	Mass*
first	$d$ (down)	$-1/3$	7
	$u$ (up)	$2/3$	3
second	$s$ (strange)	$-1/3$	120
	$c$ (charm)	$2/3$	1200
third	$b$ (bottom)	$-1/3$	4300
	$t$ (top)	$2/3$	174000

\*Light quark masses are imprecise and speculative; for effective masses in mesons and baryons, see Chapter 5.

### Mediators (spin 1)

Force	Mediator	Charge	Mass*	Lifetime	Principal Decays
Strong	$g$ (8 gluons)	0	0	$\infty$	$-$
Electromagnetic	$\gamma$ (photon)	0	0	$\infty$	$-$
Weak	$W^\pm$ (charged)	$\pm 1$	80,420	$3.11 \times 10^{-25}$	$e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, cX \rightarrow \text{hadrons}$
	$Z^0$ (neutral)	0	91,190	$2.64 \times 10^{-25}$	$e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q} \rightarrow \text{hadrons}$

### Baryons (spin 1/2)

Baryon	Quark Content	Charge	Mass	Lifetime	Principal Decays
$N \begin{cases} p \\ n \end{cases}$	$uud$ $udd$	1 0	938.272 939.565	$\infty$ 885.7	— $p e \bar{\nu}_e$
$\Lambda$	$uds$	0	1115.68	$2.63 \times 10^{-10}$	$p\pi^-, n\pi^0$
$\Sigma^+$	$uus$	1	1189.37	$8.02 \times 10^{-11}$	$p\pi^0, n\pi^+$
$\Sigma^0$	$uds$	0	1192.64	$7.4 \times 10^{-20}$	$\Lambda\gamma$
$\Sigma^-$	$dds$	-1	1197.45	$1.48 \times 10^{-10}$	$n\pi^-$
$\Xi^0$	$uss$	0	1314.8	$2.90 \times 10^{-10}$	$\Lambda\pi^0$
$\Xi^-$	$dss$	-1	1321.3	$1.64 \times 10^{-10}$	$\Lambda\pi^-$
$\Lambda_c^+$	$udc$	1	2286.5	$2.00 \times 10^{-13}$	$pK\pi, \Lambda\pi\pi, \Sigma\pi\pi$

### Baryons (spin 3/2)

Baryon	Quark Content	Charge	Mass	Lifetime	Principal Decays
$\Delta$	$uuu, uud, udd, ddd$	2, 1, 0, -1	1232	$5.6 \times 10^{-24}$	$N\pi$
$\Sigma^*$	$uus, uds, dds$	1, 0, -1	1385	$1.8 \times 10^{-23}$	$\Lambda\pi, \Sigma\pi$
$\Xi^*$	$uss, dss$	0, -1	1533	$6.9 \times 10^{-23}$	$\Xi\pi$
$\Omega^-$	$sss$	-1	1672	$8.2 \times 10^{-11}$	$\Lambda K^-, \Xi\pi$

### Pseudoscalar Mesons (spin 0)

Meson	Quark Content	Charge	Mass	Lifetime	Principal Decays
$\pi^\pm$	$u\bar{d}, d\bar{u}$	1, -1	139.570	$2.60 \times 10^{-8}$	$\mu\nu_\mu$
$\pi^0$	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	134.977	$8.4 \times 10^{-17}$	$\gamma\gamma$
$K^\pm$	$u\bar{s}, s\bar{u}$	1, -1	493.68	$1.24 \times 10^{-8}$	$\mu\nu_\mu, \pi\pi, \pi\pi\pi$
$K^0, \bar{K}^0$	$d\bar{s}, s\bar{d}$	0	497.65	$\begin{cases} K_S^0: 8.95 \times 10^{-11} \\ K_L^0: 5.11 \times 10^{-8} \end{cases}$	$\pi\pi$ $\pi e \nu_e, \pi\mu\nu_\mu, \pi\pi\pi$
$\eta$	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	0	547.51	$5.1 \times 10^{-19}$	$\gamma\gamma, \pi\pi\pi$
$\eta'$	$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$	0	957.78	$3.2 \times 10^{-21}$	$\eta\pi\pi, \rho\gamma$
$D^\pm$	$c\bar{d}, d\bar{c}$	1, -1	1869.3	$1.04 \times 10^{-12}$	$K\pi\pi, K\mu\nu_\mu, K e \nu_e$
$D^0, \bar{D}^0$	$c\bar{u}, u\bar{c}$	0	1864.5	$4.1 \times 10^{-13}$	$K\pi\pi, K e \nu_e, K\mu\nu_\mu$
$D_s^\pm$	$c\bar{s}, s\bar{c}$	1, -1	1968.2	$5.0 \times 10^{-13}$	$\eta\rho, \phi\pi\pi, \phi\rho$
$B^\pm$	$u\bar{b}, b\bar{u}$	1, -1	5279.0	$1.6 \times 10^{-12}$	$D^*\ell\nu_\ell, D\ell\nu_\ell, D^*\pi\pi\pi$
$B^0, \bar{B}^0$	$d\bar{b}, b\bar{d}$	0	5279.4	$1.5 \times 10^{-12}$	$D^*\ell\nu_\ell, D\ell\nu_\ell, D^*\pi\pi$

### Vector Mesons (spin 1)

Meson	Quark Content	Charge	Mass	Lifetime	Principal Decays
$\rho$	$u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}$	1, 0, -1	775.5	$4 \times 10^{-24}$	$\pi\pi$
$K^*$	$u\bar{s}, d\bar{s}, s\bar{d}, s\bar{u}$	1, 0, -1	894	$1 \times 10^{-23}$	$K\pi$
$\omega$	$(u\bar{u} + d\bar{d})/\sqrt{2}$	0	782.6	$8 \times 10^{-23}$	$\pi\pi\pi, \pi\gamma$
$\phi$	$s\bar{s}$	0	1019	$2 \times 10^{-22}$	$KK$
$\psi$	$c\bar{c}$	0	3097	$7 \times 10^{-21}$	$e^+e^-, \mu^+\mu^-, 5\pi, 7\pi$
$D^*$	$c\bar{d}, c\bar{u}, u\bar{c}, d\bar{c}$	1, 0, -1	2008	$3 \times 10^{-21}$	$D\pi, D\gamma$
$\Upsilon$	$b\bar{b}$	0	9460	$1 \times 10^{-20}$	$e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

## Spin 1/2

**Pauli Matrices:**

$$\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk} \sigma_k, \quad (\mathbf{a} \cdot \boldsymbol{\sigma})(\mathbf{b} \cdot \boldsymbol{\sigma}) = \mathbf{a} \cdot \mathbf{b} + i\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b})$$

$$\sigma_i^\dagger = \sigma_i = \sigma_i^{-1}, \quad e^{i\boldsymbol{\theta} \cdot \boldsymbol{\sigma}} = \cos \theta + i(\hat{\boldsymbol{\theta}} \cdot \boldsymbol{\sigma}) \sin \theta$$

**Dirac Matrices:**

$$\gamma^0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i \equiv \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^{0\dagger} = \gamma^0, \quad \gamma^{i\dagger} = -\gamma^i, \quad \gamma^0 \gamma^\mu \dagger \gamma^0 = \gamma^\mu$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \{\gamma^\mu, \gamma^5\} = 0, \quad (\gamma^5)^2 = 1$$

(For product rules and trace theorems see Appendix C.)

**Dirac Equation:**

$$i\hbar \gamma^\mu \partial_\mu \psi - mc \psi = 0$$

$$(\not{\partial} - mc)u = 0, \quad (\not{\partial} + mc)v = 0, \quad \bar{u}(\not{\partial} - mc) = 0, \quad \bar{v}(\not{\partial} + mc) = 0$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0, \quad \bar{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0, \quad \not{a} \equiv a_\mu \gamma^\mu$$

## Feynman Rules

	External Lines	Propagators
Spin 0:	Nothing	$\frac{i}{q^2 - (mc)^2}$
Spin 1/2:	$\begin{cases} \text{Incoming particle:} & u \\ \text{Incoming antiparticle:} & \bar{v} \\ \text{Outgoing particle:} & \bar{u} \\ \text{Outgoing antiparticle:} & v \end{cases}$	$\frac{i(\not{q} + mc)}{q^2 - (mc)^2}$

$$\text{Spin 1: } \begin{cases} \text{Incoming: } \epsilon_\mu \\ \text{Outgoing: } \epsilon_\mu^* \end{cases} \begin{cases} \text{Massless: } \frac{-ig_{\mu\nu}}{q^2} \\ \text{Massive: } \frac{-i[g_{\mu\nu} - q_\mu q_\nu / (mc)^2]}{q^2 - (mc)^2} \end{cases}$$

(For vertex factors see Appendix D.)

## Fundamental Constants

Planck's constant:	$\hbar$	$= 1.05457 \times 10^{-34} \text{ J s}$ $= 6.58212 \times 10^{-22} \text{ MeV s}$
Speed of light:	$c$	$= 2.99792 \times 10^8 \text{ m/s}$
Mass of electron:	$m_e$	$= 9.10938 \times 10^{-31} \text{ kg} = 0.510999 \text{ MeV}/c^2$
Mass of proton:	$m_p$	$= 1.67262 \times 10^{-27} \text{ kg} = 938.272 \text{ MeV}/c^2$
Electron charge (magnitude):	$e$	$= 1.60218 \times 10^{-19} \text{ C}$ $= 4.80320 \times 10^{-10} \text{ esu}$
Fine structure constant:	$\alpha$	$= e^2/\hbar c = 1/137.036$
Bohr radius:	$a$	$= \hbar^2/m_e e^2 = 5.29177 \times 10^{-11} \text{ m}$
Bohr energies:	$E_n$	$= -m_e e^4 / 2\hbar^2 n^2 = -13.6057/n^2 \text{ eV}$
Classical electron radius:	$r_e$	$= e^2/m_e c^2 = 2.81794 \times 10^{-15} \text{ m}$
QED coupling constant:	$g_e$	$= e\sqrt{4\pi/\hbar c} = 0.302822$
Weak coupling constants:	$g_w$	$= g_e/\sin\theta_w = 0.6295;$
	$g_z$	$= g_w/\cos\theta_w = 0.7180$
Weak mixing angle:	$\theta_w$	$= 28.76^\circ \quad (\sin^2\theta_w = 0.2314)$
Strong coupling constant:	$g_s$	$= 1.214$

## Conversion Factors

1 Å	$= 0.1 \text{ nm} = 10^{-10} \text{ m}$
1 fm	$= 10^{-15} \text{ m}$
1 barn	$= 10^{-28} \text{ m}^2$
1 eV	$= 1.60218 \times 10^{-19} \text{ J}$
1 MeV/ $c^2$	$= 1.78266 \times 10^{-30} \text{ kg}$
1 Coulomb	$= 2.99792 \times 10^9 \text{ esu}$

## Introduction

### Elementary Particle Physics

Elementary particle physics addresses the question, ‘What is matter made of?’ at the most fundamental level – which is to say, on the smallest scale of size. It’s a remarkable fact that matter at the subatomic level consists of tiny chunks, with vast empty spaces in between. Even more remarkable, these tiny chunks come in a small number of different types (electrons, protons, neutrons, pi mesons, neutrinos, and so on), which are then replicated in astronomical quantities to make all the ‘stuff’ around us. And these replicas are absolutely perfect copies – not just ‘pretty similar’, like two Fords coming off the same assembly line, but utterly *indistinguishable*. You can’t stamp an identification number on an electron, or paint a spot on it – if you’ve seen one, you’ve seen them all. This quality of absolute identicalness has no analog in the macroscopic world. (In quantum mechanics it is reflected in the Pauli exclusion principle.) It enormously simplifies the task of elementary particle physics: we don’t have to worry about big electrons and little ones, or new electrons and old ones – an electron is an electron is an electron. It didn’t have to be so easy.

My first job, then, is to introduce you to the various kinds of elementary particles – the actors, if you will, in the drama. I could simply *list* them, and tell you their properties (mass, electric charge, spin, etc.), but I think it is better in this case to adopt a historical perspective, and explain how each particle first came on the scene. This will serve to endow them with character and personality, making them easier to remember and more interesting to watch. Moreover, some of the stories are delightful in their own right.

Once the particles have been introduced, in Chapter 1, the issue becomes, ‘How do they interact with one another?’ This question, directly or indirectly, will occupy us for the rest of the book. If you were dealing with two *macroscopic* objects, and you wanted to know how they interact, you would probably begin by holding them at various separation distances and measuring the force between them. That’s how Coulomb determined the law of electrical repulsion between two charged pith balls, and how Cavendish measured the gravitational attraction of two lead weights. But you can’t pick up a proton with tweezers or tie an electron onto the end of a piece of string; they’re just too small. For practical reasons, therefore, we have to resort to

less direct means to probe the interactions of elementary particles. As it turns out, almost all of our experimental information comes from three sources: (1) scattering events, in which we fire one particle at another and record (for instance) the angle of deflection; (2) decays, in which a particle spontaneously disintegrates and we examine the debris; and (3) bound states, in which two or more particles stick together, and we study the properties of the composite object. Needless to say, determining the interaction law from such indirect evidence is not a trivial task. Ordinarily, the procedure is to *guess* a form for the interaction and compare the resulting theoretical predictions with the experimental data.

The formulation of such a guess ('model' is a more respectable term for it) is guided by certain general principles, in particular, special relativity and quantum mechanics. In the diagram below I have sketched out four realms of mechanics:

	Small →	
Fast ↓	Classical mechanics	Quantum mechanics
	Relativistic mechanics	Quantum field theory

The world of everyday life, of course, is governed by classical mechanics. But for objects that travel very fast (at speeds comparable to  $c$ ), the classical rules are modified by special relativity, and for objects that are very small (comparable to the size of atoms, roughly speaking), classical mechanics is superseded by quantum mechanics. Finally, for things that are both fast *and* small, we require a theory that incorporates relativity and quantum principles: quantum field theory. Now, elementary particles *are* extremely small, of course, and typically they are also very fast. So, elementary particle physics naturally falls under the dominion of quantum field theory.

Please observe the distinction here between a *type of mechanics* and a *particular force law*. Newton's law of universal gravitation, for example, describes a specific interaction (gravity), whereas Newton's three laws of motion define a mechanical system (classical mechanics), which (within its jurisdiction) governs *all* interactions. The force law tells you what  $F$  is, in the case at hand; the mechanics tells you how to *use*  $F$  to determine the motion. The goal of elementary particle dynamics, then, is to guess a set of force laws which, within the context of quantum field theory, correctly describe particle behavior.

However, some general features of this behavior have nothing to do with the detailed form of the interactions. Instead they follow directly from relativity, from quantum mechanics, or from the combination of the two. For example, in relativity, energy and momentum are always conserved, but (rest) mass is not. Thus the decay  $\Delta \rightarrow p + \pi$  is perfectly acceptable, even though the  $\Delta$  weighs more than the sum of  $p$  plus  $\pi$ . Such a process would not be possible in classical mechanics, where mass is strictly conserved. Moreover, relativity allows for particles of zero (rest) mass – the very idea of a massless particle is nonsense in classical mechanics – and as we shall see, photons and gluons *are* massless.



In quantum mechanics a physical system is described by its *state*,  $s$  (represented by the wave function  $\psi_s$  in Schrödinger's formulation, or by the *ket*  $|s\rangle$  in Dirac's theory). A physical process, such as scattering or decay, consists of a *transition* from one state to another. But in quantum mechanics the outcome is not uniquely determined by the initial conditions; all we can hope to calculate, in general, is the *probability* for a given transition to occur. This indeterminacy is reflected in the observed behavior of particles. For example, the charged pi meson ordinarily disintegrates into a muon plus a neutrino, but occasionally one will decay into an *electron* plus a neutrino. There's no difference in the original pi mesons; they're all identical. It is simply a fact of nature that a given particle can go either way.

Finally, the union of relativity and quantum mechanics brings certain extra dividends that neither one can offer by itself: the existence of antiparticles (with the same mass and lifetime as the particle itself, but opposite electric charge), a proof of the Pauli exclusion principle (which in nonrelativistic quantum mechanics is simply an ad hoc hypothesis), and the so-called *TCP theorem*. I'll tell you more about these later on; my purpose in mentioning them here is to emphasize that these are features of the mechanical system itself, not of the particular model. Short of a catastrophic revolution, they are untouchable. By the way, quantum field theory in all its glory is difficult and deep, but don't be alarmed: Feynman invented a beautiful and intuitively satisfying formulation that is not hard to learn; we'll come to that in Chapter 6. (The *derivation* of Feynman's rules from the underlying quantum field theory is a different matter, which can easily consume the better part of an advanced graduate course, but this need not concern us here.)

In the 1960s and 1970s a theory emerged that described all of the known elementary particle interactions, except gravity. (As far as we can tell, gravity is much too weak to play any significant role in ordinary particle processes.) This theory – or, more accurately, this collection of related theories, based on two families of elementary particles (quarks and leptons), and incorporating quantum electrodynamics, the Glashow–Weinberg–Salam theory of electroweak processes, and quantum chromodynamics – has come to be called the *Standard Model*. No one pretends that it is the final word on the subject, but at least we are now playing with a full deck of cards. Since 1978, when the Standard Model achieved the status of 'orthodoxy', it has met every experimental test. Moreover, it has an attractive aesthetic feature: all of the fundamental interactions derive from one general principle, the requirement of *local gauge invariance*. It seems certain that future developments will involve extensions of the Standard Model, not its repudiation. This book might be called an 'Introduction to the Standard Model'.

As that alternative title suggests, it is a book about elementary particle *theory*, with very little on experimental methods or instrumentation. These are important matters, and an argument can be made for integrating them into a text such as this, but they can also be distracting, and interfere with the clarity and elegance of the theory itself. I encourage you to read about the experimental aspects of the subject, and from time to time I will refer you to particularly accessible accounts. But for now, I'll confine myself to scandalously brief answers to the two most obvious experimental questions.

## How Do You *Produce* Elementary Particles?

Electrons and protons are no problem; these are the stable constituents of ordinary matter. To produce electrons one simply heats up a piece of metal, and they come boiling off. If you want a *beam* of electrons, you just set up a positively charged plate nearby, to attract them over, and poke a small hole in it; the electrons that make it through the hole constitute the beam. Such an *electron gun* is the starting element in a television tube or an oscilloscope or an electron accelerator (Figure I.1).

To obtain protons you ionize hydrogen (in other words, strip off the electron). In fact, if you're using the protons as a *target*, you don't even need to bother about the electrons; they're so light that an energetic incident particle will knock them out of the way. Thus, a tank of hydrogen is essentially a tank of protons. For more exotic particles there are three main sources: cosmic rays, nuclear reactors, and particle accelerators.

- *Cosmic rays*: The earth is constantly bombarded with high-energy particles (principally protons) coming from outer space. What the source of these particles might be remains something of a mystery; at any rate, when they hit atoms in the upper atmosphere they produce showers of secondary particles (mostly muons and neutrinos, by the



**Fig. I.1** SLAC; the straight line is the accelerator itself.  
(Courtesy Stanford Linear Accelerator Center.)

time they reach ground level), which rain down on us all the time. As a source of elementary particles, cosmic rays have two virtues: they are free, and their energies can be enormous – far greater than we could possibly produce in the laboratory. But they have two major disadvantages: the rate at which they strike any detector of reasonable size is very low, and they are completely uncontrollable. So cosmic ray experiments call for patience and luck.

- *Nuclear reactors:* When a radioactive nucleus disintegrates, it may emit a variety of particles – neutrons, neutrinos, and what used to be called alpha rays (actually, alpha *particles*, which are bound states of two neutrons plus two protons), beta rays (actually, electrons or positrons), and gamma rays (actually, photons).
- *Particle accelerators:* You start with electrons or protons, accelerate them to high energy, and smash them into a target (Figure I.1). By skillful arrangements of absorbers and magnets, you can separate the particle species that you wish to study from the resulting debris. Nowadays it is possible in this way to generate intense secondary beams of positrons, muons, pions, kaons, *B*-mesons, antiprotons, and neutrinos, which in turn can be fired at another target. The stable particles – electrons, protons, positrons, and antiprotons – can even be fed into giant *storage rings* in which, guided by powerful magnets, they circulate at high speed for hours at a time, to be extracted and used at the required moment [1].

In general, the heavier the particle you want to produce, the higher must be the energy of the collision. That's why, historically, lightweight particles tend to be discovered first, and as time goes on, and accelerators become more powerful, heavier and heavier particles are found. It turns out that you gain enormously in *relative* energy if you collide two high-speed particles head-on, as opposed to firing one particle at a stationary target. (Of course, this calls for much better aim!) For this reason many contemporary experiments involve colliding beams from intersecting storage rings; if the particles miss on the first pass, they can try again the next time around. Indeed, with electrons and positrons (or protons and antiprotons) the *same ring* can be used, with the plus charges circulating in one direction and minus charges the other. Unfortunately, when a charged particle accelerates it radiates, thereby losing energy. In the case of circular motion (which, of course, involves acceleration) this is called *synchrotron radiation*, and it severely limits the efficiency of storage rings for energetic electrons (heavier particles with the same energy accelerate less, so synchrotron radiation is not such a problem for them). For this reason electron scattering experiments will increasingly turn to *linear* colliders, while storage rings will continue to be used for protons and heavier particles.

There is another reason why particle physicists are always pushing for higher energies: in general, the higher the energy of the collision, the closer the two particles come to one another. So if you want to study an interaction at very short range, you need very energetic particles. In quantum-mechanical terms, a particle of momentum  $p$  has an associated wavelength  $\lambda$  given by the de Broglie formula  $\lambda = h/p$ , where  $h$  is Planck's constant. At large wavelengths (low momenta) you can only hope to resolve relatively large structures; in order to examine something extremely small, you need comparably short wavelengths, and hence high momenta. If you like, consider this a manifestation of the uncertainty principle ( $\Delta x \Delta p \geq h/4\pi$ ) – to make  $\Delta x$  small,  $\Delta p$  must be large. However you look at it, the conclusion is the same: to probe *small distances* you need *high energies*.

At present the most powerful accelerator in the world is the *Tevatron* at Fermilab (Figure I.2), with a maximum beam energy of almost 1 TeV. The *tevatron* (a proton–antiproton collider) began operation in 1983; its successor, the Superconducting Supercollider (SSC) was under construction in 1993 when the project was terminated by Congress. As a result, there has been a long period in which no fundamental progress was possible. This dry spell should end in 2008, when the Large Hadron Collider (LHC) at CERN starts taking data (Figure I.3). The LHC is designed to reach beam energies in excess of 7 TeV, and the hope is that this new terrain will include the Higgs particle, possibly supersymmetric particles, and – best of all – something completely unexpected [2]. It's not clear what comes after the LHC – most likely the proposed International Linear Collider (ILC). But, accelerators have become so huge (the SSC would have been 87 km in circumference) that there is not much room for expansion. Perhaps we are approaching the end



**Fig. I.2** Fermilab; the large circle in the background is the Tevatron. (Courtesy Fermilab Visual Media Services.)





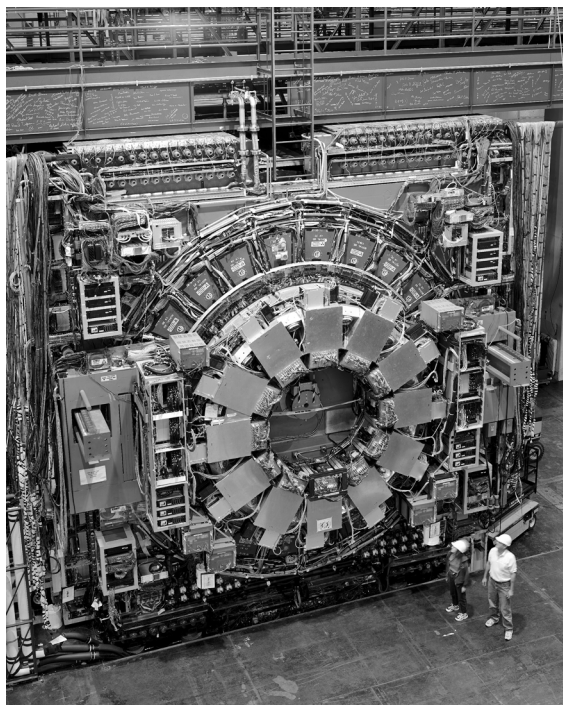
**Fig. I.3** CERN; the circle indicates the path of the LHC tunnel (formerly LEP) – Geneva and Mt Blanc are in the background. (Courtesy CERN.)

of the accelerator era, and particle physicists will have to turn to astrophysics and cosmology for information about higher energies. Or perhaps someone will have a clever new idea for squeezing energy onto an elementary particle.\*

### How Do You *Detect* Elementary Particles?

There are many kinds of particle detectors – Geiger counters, cloud chambers, bubble chambers, spark chambers, drift chambers, photographic emulsions, Čerenkov counters, scintillators, photomultipliers, and so on. Actually, a typical modern

\* In macroscopic terms the amount of energy involved is not that great – after all, 1 TeV ( $10^{12}$  eV) is only  $10^{-7}$  Joules; the problem is how to *deliver* that energy to a particle. No law of physics prevents you from doing so, but nobody has yet figured out a way to do it without gigantic (and expensive) machinery.



**Fig. I.4** The CDF detector at Fermilab, where the top quark was discovered. (Courtesy Fermilab Visual Media Services.)

detector has whole arrays of these devices, wired up to a computer that tracks the particles and displays their trajectories on a television screen (Figure I.4). The details do not concern us, but there is one thing you should be aware of: most detection mechanisms rely on the fact that when high-energy charged particles pass through matter they ionize atoms along their path. The ions then act as ‘seeds’ in the formation of droplets (cloud chamber) or bubbles (bubble chamber) or sparks (spark chamber), as the case may be. But electrically *neutral* particles do not cause ionization, and they leave no tracks. For instance, if you look at the bubble chamber photograph in Figure 1.9, you will see that the five neutral particles are ‘invisible’; their paths have been reconstructed by analyzing the tracks of the *charged* particles in the picture and invoking conservation of energy and momentum at each vertex. Notice also that most of the tracks in the picture are *curved* (actually, *all* of them are, to some extent; try holding a ruler up to one you think is straight). The bubble chamber was placed between the poles of a giant magnet; in a magnetic field  $B$ , a particle of charge  $q$  and momentum  $p$  will move in a circle of radius  $R$  given by the famous *cyclotron formula*:  $R = pc/qB$ , where  $c$  is the speed of light. The curvature of the track in a known magnetic field thus affords a very simple measure of the particle’s momentum. Moreover, we can immediately tell the *sign* of the charge from the *direction* of the curve.

## Units

Elementary particles are small, so for our purposes the *normal* mechanical units – grams, ergs, joules, and so on – are inconveniently large. Atomic physicists introduced the *electron volt* – the energy acquired by an electron when accelerated through a potential difference of 1 volt:  $1 \text{ eV} = 1.6 \times 10^{-19}$  joules. For us the eV is inconveniently *small*, but we're stuck with it. Nuclear physicists use keV ( $10^3$  eV); typical energies in particle physics are MeV ( $10^6$  eV), GeV ( $10^9$  eV), or even TeV ( $10^{12}$  eV). Momenta are measured in MeV/ $c$  (or GeV/ $c$ , or whatever), and masses in MeV/ $c^2$ . Thus the proton weighs  $938 \text{ MeV}/c^2 = 1.67 \times 10^{-24}$  g.

Actually, particle theorists are lazy (or clever, depending on your point of view) – they seldom include the  $c$ 's and  $\hbar$ 's ( $\hbar \equiv h/2\pi$ ) in their formulas. You're just supposed to fit them in for yourself at the end, to make the dimensions come out right. As they say in the business, 'set  $c = \hbar = 1$ '. This amounts to working in units such that time is measured in centimeters and mass and energy in inverse centimeters; the unit of time is the time it takes light to travel 1 cm, and the unit of energy is the energy of a photon whose wavelength is  $2\pi$  cm. Only at the end of the problem do we revert to conventional units. This makes everything look very elegant, but I thought it would be wiser in this book to keep all the  $c$ 's and  $\hbar$ 's where they belong, so that you can check for dimensional consistency as you go along. (If this offends you, remember that it is easier for you to ignore an  $\hbar$  you don't like than for someone else to conjure one up in just the right place.)

Finally, there is the question of what units to use for electric charge. In introductory physics courses most instructors favor the *SI* system, in which charge is measured in *coulombs*, and Coulomb's law reads

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (\text{SI})$$

Most advanced work is done in the *Gaussian* system, in which charge is measured in *electrostatic units* (esu), and Coulomb's law is written

$$F = \frac{q_1 q_2}{r^2} \quad (\text{G})$$

But elementary particle physicists prefer the *Heaviside – Lorentz* system, in which Coulomb's law takes the form

$$F = \frac{1}{4\pi} \frac{q_1 q_2}{r^2} \quad (\text{HL})$$

The three units of charge are related as follows:

$$q_{\text{HL}} = \sqrt{4\pi} q_{\text{G}} = \frac{1}{\sqrt{\epsilon_0}} q_{\text{SI}}$$

In this book I shall use Gaussian units exclusively, in order to avoid unnecessary confusion in an already difficult subject. Whenever possible I will express results in terms of the *fine structure constant*

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036}$$

where  $e$  is the charge of the electron in Gaussian units. Most elementary particle texts write this as  $e^2/4\pi$ , because they are measuring charge in Heaviside–Lorentz units and setting  $c = \hbar = 1$ ; but everyone agrees that the *number* is  $1/137$ .

### Further reading

Since the early 1960s, the Particle Data Group at Berkeley has periodically issued a listing of the established particles and their properties. These are published every other year in *Reviews of Modern Physics* or *Journal of Physics G*, and summarized in a (free) booklet that can be ordered on the web at <http://pdg.lbl.gov>. In the early days this summary took the form of ‘wallet cards’, but by 2006 it had grown to a densely packed 315 pages. I shall refer to it as the *Particle Physics Booklet* (PPB). Every student of elementary particle physics must have a copy – don’t leave home without it! The longer version, called the *Review of Particle Physics* (RPP) is the bible for professionals – the 2006 edition runs to 1231 pages, and it includes authoritative articles on every relevant subject, written by the world’s leading experts [3]. If you want the definitive, up-to-date word on any particular topic, this is the place to go (it is also available on-line, at the Particle Data Group web site).

Particle physics is an enormous and rapidly changing subject. My aim in this book is to introduce you to some important ideas and methods, to give you a sense of what’s out there to be learned, and perhaps to stimulate your appetite for more. If you want to read further in quantum field theory, I particularly recommend:

- Bjorken, J. D. and Drell, S. D. (1964) *Relativistic Quantum Mechanics and Relativistic Quantum Fields*, McGraw-Hill, New York.
- Itzykson, C. and Zuber, J.-B. (1980) *Quantum Field Theory*, McGraw-Hill, New York.
- Peskin, M. E. and Schroeder, D. V. (1995) *An Introduction to Quantum Field Theory*, Perseus, Cambridge, MA.
- Ryder, L. H. (1985) *Quantum Field Theory*, Cambridge University Press, Cambridge, UK.
- Sakurai, J. J. (1967) *Advanced Quantum Mechanics*, Addison-Wesley, Reading, MA.

I warn you, however, that these are all difficult and advanced books.



For elementary particle physics itself, the following books (listed in order of increasing difficulty) are especially useful:

- Close, F., Marten, M. and Sutton, C. (1987) *The Particle Explosion*, Oxford University Press, Oxford, UK.
- Frauenfelder, H. and Henley, E. M. (1991) *Subatomic Physics*, 2nd edn, Prentice-Hall, Englewood Cliffs, NJ.
- Gottfried, K. and Weisskopf, V. F. (1984) *Concepts of Particle Physics*, Oxford University Press, Oxford.
- Perkins, D. H. (2000) *Introduction to High-Energy Physics*, 4th Ed, Cambridge University Press, Cambridge, UK.
- Halzen, F. and Martin, A. D. (1984) *Quarks and Leptons*, John Wiley & Sons, Ltd, New York.
- Roe, B. P. (1996) *Particle Physics at the New Millennium*, Springer, New York.
- Aitchison, I. J. R. and Hey, A. J. G. (2003) *Gauge Theories in Particle Physics*, 3rd edn, Institute of Physics, Bristol, UK.
- Seiden, A. (2005) *Particle Physics: A Comprehensive Introduction*, Addison-Wesley, San Francisco, CA.
- Quigg, C. (1997) *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*, Addison-Wesley, Reading, MA.

## References

- 1 For a comprehensive bibliography see Chao, Q. W. (2006) *American Journal of Physics*, **74**, 855.
  - 2 Smith, C. L. (July 2000) *Scientific American*, 71; (a) Lederman, L. (2007) *Nature*, **448**, 310.
  - 3 The current reference is Yao, W.-M. *et al.* (2006) *Journal of Physics*, **G 33**, 1.
1. But because you will want to use the most recent versions, I will simply refer to them as *Particle Physics Booklet* and *Review of Particle Physics*, appending the year when it is relevant.

