Logic FOR DUMMIES®

by Mark Zegarelli

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About the Author

Mark Zegarelli is a professional writer with degrees in both English and Math from Rutgers University. He has earned his living for many years writing vast quantities of logic puzzles, a hefty chunk of software documentation, and the occasional book or film review. Along the way, he's also paid a few bills doing housecleaning, decorative painting, and (for ten hours) retail sales. He likes writing best, though.

Mark lives mostly in Long Branch, New Jersey, and sporadically in San Francisco, California.

Dedication

This is for Mark Dembrowski, with love for his unfailing support, encouragement, and wisdom.

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Introduction

You use logic every day — and I bet you didn't even realize it. For instance, consider these examples of times when you might use logic:

- Planning an evening out with a friend
- Asking your boss for a day off or for a raise
- Picking out a shirt to buy among several that you like
- Explaining to your kids why homework comes before TV

At all of these times, you use logic to clarify your thinking and get other people to see things from your perspective.

Even if you don't always act on it, logic is natural — at least to humans. And logic is one of the big reasons why humans have lasted so long on a planet filled with lots of other creatures that are bigger, faster, more numerous, and more ferocious.

And because logic is already a part of your life, after you notice it, you'll see it working (or *not* working) everywhere you look.

This book is designed to show you how logic arises naturally in daily life. Once you see that, you can refine certain types of thinking down to their essence. Logic gives you the tools for working with what you already know (the premises) to get you to the next step (the conclusion). Logic is also great for helping you spot the flaws in arguments — unsoundness, hidden assumptions, or just plain unclear thinking.

About This Book

Logic has been around a long time — almost 2,400 years and counting! So, with so many people (past and present) thinking and writing about logic, you may find it difficult to know where to begin. But, never fear, I wrote this book with you in mind.

If you're taking an introductory course in logic, you can supplement your knowledge with this book. Just about everything you're studying in class is explained here simply, with lots of step-by-step examples. At the same time, if you're just interested in seeing what logic is all about, this book is also a great place for you to start.

Logic For Dummies is for anybody who wants to know about logic — what it is, where it came from, why it was invented, and even where it may be going. If you're taking a course in logic, you'll find the ideas that you're studying explained clearly, with lots of examples of the types of problems your professor will ask you to do. In this book, I give you an overview of logic in its many forms and provide you with a solid base of knowledge to build upon.

Logic is one of the few areas of study taught in two different college departments: math and philosophy. The reason that logic can fit into two seemingly different categories is historical: Logic was founded by Aristotle and developed by philosophers for centuries. But, about 150 years ago, mathematicians found that logic was an indispensable tool for grounding their work as it became more and more abstract.

One of the most important results of this overlap is formal logic, which takes ideas from philosophical logic and applies them in a mathematical framework. Formal logic is usually taught in philosophy departments as a purely computational (that is, mathematical) pursuit.

When writing this book, I tried to balance both of these aspects of logic. Generally speaking, the book begins where logic began — with philosophy — and ends where it has been taken — in mathematics.

Conventions Used in This Book

To help you navigate through this book, we use the following conventions:

- *Italics* are used for emphasis and to highlight new words and terms defined in the text. They're also used for variables in equations.
- **Boldfaced** text indicates keywords in bulleted lists and also true (**T**) and false (**F**) values in equations and tables. It's also used for the 18 rules of inference in SL and the 5 rules of inference in QL.
- Sidebars are shaded gray boxes that contain text that's interesting to know but not critical to your understanding of the chapter or topic.
- Twelve-point boldfaced text (**T** and **F**) text is used in running examples of truth tables and quick tables to indicate information that's just been added. It's used in completed truth tables and quick tables to indicate the truth value of the entire statement.
- Parentheses are used throughout statements, instead of a combination of parentheses, brackets, and braces. Here's an example:

  $$\sim((P \vee Q) \rightarrow \sim R)$$

What You're Not to Read

I would be thrilled if you sat down and read this book from cover to cover, but let's face it: No one has that kind of time these days. How much of this book you read depends on how much logic you already know and how thoroughly you want to get into it.

Do, however, feel free to skip anything marked with a Technical Stuff icon. This info, although interesting, is usually pretty techie and very skippable. You can also bypass any sidebars you see. These little asides often provide some offbeat or historical info, but they aren't essential to the other material in the chapter.

Foolish Assumptions

Here are a few things we've assumed about you:

- You want to find out more about logic, whether you're taking a course or just curious.
- You can distinguish between true and false statements about commonly known facts, such as "George Washington was the first president," and "The Statue of Liberty is in Tallahassee."
- You understand simple math equations.
- You can grasp really simple algebra, such as solving for x in the equation $7 - x = 5$

How This Book Is Organized

This book is separated into six parts. Even though each part builds on the information from earlier parts, the book is still arranged in a modular way. So, feel free to skip around as you like. For example, when I discuss a new topic that depends on more basic material, I refer you to the chapter where I introduced those basics. If, for right now, you only need info on a certain topic, check out the index or the table of contents — they'll for sure point you in the right direction.

Here's a thumbnail sketch of what the book covers:

Part I: Overview of Logic

What is logic? What does it mean to think logically, or for that matter illogically, and how can you tell? Part I answers these questions (and more!). The chapters in this part discuss the structure of a logical argument, explain what premises and conclusions are, and track the development of logic in its many forms, from the Greeks all the way to the Vulcans.

Part II: Formal Sentential Logic (SL)

Part II is your introduction to formal logic. Formal logic, also called symbolic logic, uses its own set of symbols to take the place of sentences in a natural language such as English. The great advantage of formal logic is that it's an easy and clear way to express logical statements that would be long and complicated in English (or Swahili).

You discover sentential logic (SL for short) and the five logical operators that make up this form. I also show how to translate back and forth between English and SL. Finally, I help you understand how to evaluate a statement to decide whether it's true or false using three simple tools: truth tables, quick tables, and truth trees.

Part III: Proofs, Syntax, and Semantics in SL

Just like any logic geek, I'm sure you're dying to know how to write proofs in SL — yeah, those pesky formal arguments that link a set of premises to a conclusion using the rules of inference. Well, you're in luck. In this part, you discover the ins and outs of proof writing. You also find out how to write conditional and indirect proofs, and how to attack proofs as efficiently as possible using a variety of proof strategies.

You also begin looking at SL from a wider perspective, examining it on the levels of both syntax and semantics.

You find out how to tell a statement from a string of symbols that just looks like a statement. I also discuss how the logical operators in SL allow you to build sentence functions that have one or more input values and an output value. From this perspective, you see how versatile SL is for expressing all possible sentence functions with a minimum of logical operators.

Part IV: Quantifier Logic (QL)

If you're looking to discover all there is to know about quantifier logic (or QL, for short), look no further: This part serves as your one-stop shopping introduction. QL encompasses everything from SL, but extends it in several important ways.

In this part, I show you how QL allows you to capture more intricacies of a statement in English by breaking it down into smaller parts than would be possible in SL. I also introduce the two quantification operators, which make it possible to express a wider variety of statements. Finally, I show you how to take what you already know about proofs and truth trees and put it to work in QL.

Part V: Modern Developments in Logic

The power and subtlety of logic becomes apparent as you examine the advances in this field over the last century. In this part, you see how logic made the 19th century dream of the computer a reality. I discuss how variations of post-classical logic, rooted in seemingly illogical assumptions, can be consistent and useful for describing real-world events.

I also show you how paradoxes fundamentally challenged logic at its very core. Paradoxes forced mathematicians to remove all ambiguities from logic by casting it in terms of axiom systems. Ultimately, paradoxes inspired one mathematician to harness paradox itself as a way to prove that logic has its limitations.

Part VI: The Part of Tens

Every *For Dummies* book contains a Part of Tens. Just for fun, this part of the book includes a few top-ten lists on a variety of topics: cool quotes, famous logicians, and pointers for passing exams.

Icons Used in This Book

Throughout this book, you'll find four icons that highlight different types of information:

I use this icon to point out the key ideas that you need to know. Make sure you understand the information in these paragraphs before reading on!

This icon highlights helpful hints that show you the easy way to get things done. Try them out, especially if you're enrolled in a logic course.

Don't skip these icons! They show you common errors that you want to avoid. The paragraphs that don this important icon help you recognize where these little traps are hiding so that you don't take a wrong step and fall in.

This icon alerts you to interesting, but unnecessary, trivia that you can read or skip over as you like.

Where to Go from Here

If you have some background in logic and you already have a handle on the Part I stuff, feel free to jump forward where the action is. Each part builds on the previous parts, so if you can read Part III with no problem, you probably don't need to concentrate on Parts I and II (unless of course you just want a little review).

If you're taking a logic course, you may want to read Parts III and IV carefully — you may even try to reproduce the proofs in those chapters with the book closed. Better to find out what you don't know while you're studying than while you're sweating out an exam!

If you're not taking a logic course — complete with a professor, exams, and a final grade — and you just want to discover the basics of logic, you may want to skip or simply skim the nitty-gritty examples of proofs in Parts III and IV. You'll still get a good sense of what logic is all about, but without the heavy lifting.

If you forge ahead to Parts IV and V, you're probably ready to tackle some fairly advanced ideas. If you're itching to get to some meaty logic, check out Chapter 22. This chapter on logical paradoxes has some really cool stuff to take your thinking to warp speed. Bon voyage!

Part I
Overview of Logic

"Laugh if you want, I'm just not comfortable teaching my class in Logic without wearing my lucky socks."

In this part . . .

So, let me guess, you just started your first logic class and you're scrambling around trying to discover the ins and outs of logic as quickly as possible because you have your first test in 48 hours. Or, maybe you're not scrambling at all and you're just looking for some insight to boost your understanding. Either way, you've come to the right place.

In this part, you get a firsthand look at what logic is all about. Chapter 1 gives an overview of how you (whether you know it or not) use logic all the time to turn the facts that you know into a better understanding of the world. Chapter 2 presents the history of logic, with a look at the many types of logic that have been invented over the centuries. Finally, if you're itching to get started, flip to Chapter 3 for an explanation of the basic structure of a logical argument. Chapter 3 also focuses on key concepts such as premises and conclusions, and discusses how to test an argument for validity and soundness.

Chapter 1

What Is This Thing Called Logic?

In This Chapter

- Seeing the world from a logical point of view
- Using logic to build valid arguments
- Applying the laws of thought
- Understanding the connection between math and logic

You and I live in an illogical world. If you doubt this fact, just flip on the evening news. Or really listen to the guy sitting at the next barstool. Or, better yet, spend the weekend with your in-laws.

With so many people thinking and acting illogically, why should you be any different? Wouldn't it be more sensible just to be as illogical as the rest of the human race?

Well, okay, being illogical on purpose is probably not the best idea. For one thing, how can it possibly be sensible to be illogical? For another, if you've picked this book up in the first place, you're probably not built to be illogical. Let's face it — some folks thrive on chaos (or claim to), while others don't.

In this chapter, I introduce you to the basics of logic and how it applies to your life. I tell you a few words and ideas that are key to logic. And, I touch very briefly on the connections between logic and math.

Getting a Logical Perspective

Whether you know it or not, you already understand a lot about logic. In fact, you already have a built-in logic detector. Don't believe me? Take this quick test to see whether you're logical:

Q: How many pancakes does it take to shingle a doghouse?

A: 23, because bananas don't have bones.

If the answer here seems illogical to you, that's a good sign that you're at least on your way to being logical. Why? Simply because if you can spot something that's illogical, you must have a decent sense of what actually is logical.

In this section, I start with what you *already* understand about logic (though you may not be aware of it), and build towards a foundation that will help you in your study of logic.

Bridging the gap from here to there

Most children are innately curious. They always want to know *why* everything is the way it is. And for every *because* they receive, they have one more *why*. For example, consider these common kid questions:

Why does the sun rise in the morning?

Why do I have to go to school?

Why does the car start when you turn the key?

Why do people break the law when they know they could go to jail?

When you think about it, there's a great mystery here: Even when the world doesn't make sense, why does it feel like it should?

Kids sense from an early age that even though they don't understand something, the answer must be somewhere. And they think, "If I'm here and the answer is there, what do I have to do to get there?" (Often, their answer is to bug their parents with more questions.)

Getting from here to there — from ignorance to understanding — is one of the main reasons logic came into existence. Logic grew out of an innate human need to make sense of the world and, as much as possible, gain some control over it.

Understanding cause and effect

One way to understand the world is to notice the connection between cause and effect.

As you grow from a child to an adult, you begin to piece together how one event causes another. Typically, these connections between cause and effect can be placed in an *if-statement.* For example, consider these if-statements: