Second Edition

Michael J. Crawley

# STATISTICS

## An introduction using R

WILEY

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## *An Introduction Using R*

**Second Edition**

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# *Contents*

# *Preface*

This book is an introduction to the essentials of statistical analysis for students who have little or no background in mathematics or statistics. The audience includes first- and second-year undergraduate students in science, engineering, medicine and economics, along with post-experience and other mature students who want to relearn their statistics, or to switch to the powerful new language of R.

For many students, statistics is the least favourite course of their entire time at university. Part of this is because some students have convinced themselves that they are no good at sums, and consequently have tried to avoid contact with anything remotely quantitative in their choice of subjects. They are dismayed, therefore, when they discover that the statistics course is compulsory. Another part of the problem is that statistics is often taught by people who have absolutely no idea how difficult some of the material is for non-statisticians. As often as not, this leads to a recipe-following approach to analysis, rather than to any attempt to understand the issues involved and how to deal with them.

The approach adopted here involves virtually no statistical theory. Instead, the assumptions of the various statistical models are discussed at length, and the practice of exposing statistical models to rigorous criticism is encouraged. A philosophy of model simplification is developed in which the emphasis is placed on estimating effect sizes from data, and establishing confidence intervals for these estimates. The role of hypothesis testing at an arbitrary threshold of significance like $\alpha = 0.05$ is played down. The text starts from absolute basics and assumes absolutely no background in statistics or mathematics.

As to presentation, the idea is that background material would be covered in a series of 1-hour lectures, then this book could be used as a guide to the practical sessions and for homework, with the students working on their own at the computer. My experience is that the material can be covered in 10–30 lectures, depending on the background of the students and the depth of coverage it is hoped to achieve. The practical work is designed to be covered in 10–15 sessions of about 1½ hours each, again depending on the ambition and depth of the coverage, and on the amount of one-to-one help available to the students as they work at their computers.

The R language of statistical computing has an interesting history. It evolved from the S language, which was first developed at the AT&T Bell Laboratories by Rick Becker, John Chambers and Allan Wilks. Their idea was to provide a software tool for professional statisticians who wanted to combine state-of-the-art graphics with powerful model-fitting capability. S is made up of three components. First and foremost, it is a powerful tool for statistical modelling. It enables you to specify and fit statistical models to your data, assess the goodness of fit and display the estimates, standard errors and predicted values derived

from the model. It provides you with the means to define and manipulate your data, but the way you go about the job of modelling is not predetermined, and the user is left with maximum control over the model-fitting process. Second, S can be used for data exploration, in tabulating and sorting data, in drawing scatter plots to look for trends in your data, or to check visually for the presence of outliers. Third, it can be used as a sophisticated calculator to evaluate complex arithmetic expressions, and a very flexible and general object-orientated programming language to perform more extensive data manipulation. One of its great strengths is in the way in which it deals with vectors (lists of numbers). These may be combined in general expressions, involving arithmetic, relational and transformational operators such as sums, greater-than tests, logarithms or probability integrals. The ability to combine frequently-used sequences of commands into functions makes S a powerful programming language, ideally suited for tailoring one's specific statistical requirements. S is especially useful in handling difficult or unusual data sets, because its flexibility enables it to cope with such problems as unequal replication, missing values, non-orthogonal designs, and so on. Furthermore, the open-ended style of S is particularly appropriate for following through original ideas and developing new concepts. One of the great advantages of learning S is that the simple concepts that underlie it provide a unified framework for learning about statistical ideas in general. By viewing particular models in a general context, S highlights the fundamental similarities between statistical techniques and helps play down their superficial differences. As a commercial product S evolved into S-PLUS, but the problem was that S-PLUS was very expensive. In particular, it was much too expensive to be licensed for use in universities for teaching large numbers of students. In response to this, two New Zealand-based statisticians, Ross Ihaka and Robert Gentleman from the University of Auckland, decided to write a stripped-down version of S for teaching purposes. The letter R 'comes before S', so what would be more natural than for two authors whose first initial was 'R' to christen their creation R. The code for R was released in 1995 under a General Public License, and the core team was rapidly expanded to 15 members (they are listed on the website, below). Version 1.0.0 was released on 29 February 2000. This book is written using version 3.0.1, but all the code will run under earlier releases.

There is now a vast network of R users world-wide, exchanging functions with one another, and a vast resource of packages containing data and programs. There is a useful publication called *The R Journal* (formerly *R News*) that you can read at CRAN. Make sure that you cite the R Core Team when you use R in published work; you should cite them like this:

> R Core Team (2014). *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna. Available from http://www.r-project.org/.

R is an Open Source implementation and as such can be freely downloaded. If you type CRAN into your Google window you will find the site nearest to you from which to download it. Or you can go directly to

http://cran.r-project.org

The present book has its own website at

http://www.imperial.ac.uk/bio/research/crawley/statistics

Here you will find all the data files used in the text; you can download these to your hard disk and then run all of the examples described in the text. The executable statements are shown in the text in red Courier New font. There are files containing all the commands for each chapter, so you can paste the code directly into R instead of typing it from the book. There is a series of 12 fully-worked stand-alone practical sessions covering a wide range of statistical analyses. Learning R is not easy, but you will not regret investing the effort to master the basics.

M.J. Crawley
Ascot
April 2014

# 1

# *Fundamentals*

The hardest part of any statistical work is getting started. And one of the hardest things about getting started is choosing the right kind of statistical analysis. The choice depends on the nature of your data and on the particular question you are trying to answer. The truth is that there is no substitute for experience: the way to know what to do is to have done it properly lots of times before.

The key is to understand what kind of *response* variable you have got, and to know the nature of your *explanatory* variables. The response variable is the thing you are working on: it is the variable whose variation you are attempting to understand. This is the variable that goes on the $y$ axis of the graph (the ordinate). The explanatory variable goes on the $x$ axis of the graph (the abscissa); you are interested in the extent to which variation in the response variable is associated with variation in the explanatory variable. A continuous measurement is a variable like height or weight that can take any real numbered value. A categorical variable is a *factor* with two or more *levels*: sex is a factor with two levels (male and female), and rainbow might be a factor with seven levels (red, orange, yellow, green, blue, indigo, violet).

It is essential, therefore, that you know:

- which of your variables is the response variable?
- which are the explanatory variables?
- are the explanatory variables continuous or categorical, or a mixture of both?
- what kind of response variable have you got – is it a continuous measurement, a count, a proportion, a time-at-death, or a category?

These simple keys will then lead you to the appropriate statistical method:

1. The explanatory variables (pick one of the rows):

| | |
|---|---|
| (a) All explanatory variables continuous | *Regression* |
| (b) All explanatory variables categorical | *Analysis of variance (ANOVA)* |
| (c) Some explanatory variables continuous some categorical | *Analysis of covariance (ANCOVA)* |

*Statistics: An Introduction Using R*, Second Edition. Michael J. Crawley.

2. The response variable (pick one of the rows):

| | |
|---|---|
| (a) Continuous | *Regression, ANOVA or ANCOVA* |
| (b) Proportion | *Logistic regression* |
| (c) Count | *Log linear models* |
| (d) Binary | *Binary logistic analysis* |
| (e) Time at death | *Survival analysis* |

There is a small core of key ideas that need to be understood from the outset. We cover these here before getting into any detail about different kinds of statistical model.

**Everything Varies**

If you measure the same thing twice you will get two different answers. If you measure the same thing on different occasions you will get different answers because the thing will have aged. If you measure different individuals, they will differ for both genetic and environmental reasons (nature and nurture). Heterogeneity is universal: spatial heterogeneity means that places always differ, and temporal heterogeneity means that times always differ.

Because everything varies, finding that things vary is simply not interesting. We need a way of discriminating between variation that is scientifically interesting, and variation that just reflects background heterogeneity. That is why you need statistics. It is what this whole book is about.

The key concept is the amount of variation that we would expect to occur by chance alone, when nothing scientifically interesting was going on. If we measure bigger differences than we would expect by chance, we say that the result is statistically significant. If we measure no more variation than we might reasonably expect to occur by chance alone, then we say that our result is not statistically significant. It is important to understand that this is not to say that the result is not important. Non-significant differences in human life span between two drug treatments may be massively important (especially if you are the patient involved). Non-significant is not the same as 'not different'. The lack of significance may be due simply to the fact that our replication is too low.

On the other hand, when nothing really *is* going on, then we want to know this. It makes life much simpler if we can be reasonably sure that there is no relationship between $y$ and $x$. Some students think that 'the only good result is a significant result'. They feel that their study has somehow failed if it shows that 'A has no significant effect on B'. This is an understandable failing of human nature, but it is not good science. The point is that we want to know the truth, one way or the other. We should try not to care too much about the way things turn out. This is not an amoral stance, it just happens to be the way that science works best. Of course, it is hopelessly idealistic to pretend that this is the way that scientists really behave. Scientists often want passionately that a particular experimental result will turn out to be statistically significant, so that they can get a *Nature* paper and get promoted. But that does not make it right.

### Significance

What do we mean when we say that a result is significant? The normal dictionary definitions of significant are 'having or conveying a meaning' or 'expressive; suggesting or implying deeper or unstated meaning'. But in statistics we mean something very specific indeed. We mean that 'a result was unlikely to have occurred by chance'. In particular, we mean 'unlikely to have occurred by chance if the null hypothesis was true'. So there are two elements to it: we need to be clear about what we mean by 'unlikely', and also what exactly we mean by the 'null hypothesis'. Statisticians have an agreed convention about what constitutes 'unlikely'. They say that an event is unlikely if it occurs less than 5% of the time. In general, the null hypothesis says that 'nothing is happening' and the alternative says that 'something *is* happening'.

### Good and Bad Hypotheses

Karl Popper was the first to point out that a good hypothesis was one that was capable of *rejection*. He argued that *a good hypothesis is a falsifiable hypothesis*. Consider the following two assertions:

A. there are vultures in the local park

B. there are no vultures in the local park

Both involve the same essential idea, but one is refutable and the other is not. Ask yourself how you would refute option A. You go out into the park and you look for vultures. But you do not see any. Of course, this does not mean that there are none. They could have seen you coming, and hidden behind you. No matter how long or how hard you look, you cannot refute the hypothesis. All you can say is 'I went out and I didn't see any vultures'. One of the most important scientific notions is that *absence of evidence is not evidence of absence*.

Option B is fundamentally different. You reject hypothesis B the first time you see a vulture in the park. Until the time that you *do* see your first vulture in the park, you work on the assumption that the hypothesis is true. But if you see a vulture, the hypothesis is clearly false, so you reject it.

### Null Hypotheses

The null hypothesis says 'nothing is happening'. For instance, when we are comparing two sample means, the null hypothesis is that the means of the two populations are the same. Of course, the two sample means are not identical, because everything varies. Again, when working with a graph of $y$ against $x$ in a regression study, the null hypothesis is that the slope of the relationship is zero (i.e. $y$ is not a function of $x$, or $y$ is independent of $x$). The essential point is that the null hypothesis is falsifiable. We reject the null hypothesis when our data show that the null hypothesis is sufficiently unlikely.

### *p* Values

Here we encounter a much-misunderstood topic. The $p$ value is *not* the probability that the null hypothesis is true, although you will often hear people saying this. In fact, $p$ values are

calculated *on the assumption that the null hypothesis is true*. It is correct to say that *p* values have to do with the plausibility of the null hypothesis, but in a rather subtle way.

As you will see later, we typically base our hypothesis testing on what are known as *test statistics*: you may have heard of some of these already (Student's *t*, Fisher's *F* and Pearson's chi-squared, for instance): *p* values are about the size of the test statistic. In particular, a *p* value is an estimate of the probability that a value of the test statistic, or a value more extreme than this, could have occurred by chance *when the null hypothesis is true*. Big values of the test statistic indicate that the null hypothesis is unlikely to be true. For sufficiently large values of the test statistic, we reject the null hypothesis and accept the alternative hypothesis.

Note also that saying 'we do not reject the null hypothesis' and 'the null hypothesis is true' are two quite different things. For instance, we may have failed to reject a false null hypothesis because our sample size was too low, or because our measurement error was too large. Thus, *p* values are interesting, but they do not tell the whole story: effect sizes and sample sizes are equally important in drawing conclusions. The modern practice is to state the *p* value rather than just to say 'we reject the null hypothesis'. That way, the reader can form their own judgement about the effect size and its associated uncertainty.

### Interpretation

It should be clear by this point that we can make two kinds of mistakes in the interpretation of our statistical models:

- we can reject the null hypothesis when it is true
- we can accept the null hypothesis when it is false

These are referred to as *Type I* and *Type II* errors, respectively. Supposing we knew the true state of affairs (which, of course, we seldom do). Then in tabular form:

| Null hypothesis | Actual situation | |
|---|---|---|
| | *True* | *False* |
| Accept | Correct decision | Type II |
| Reject | Type I | Correct decision |

### Model Choice

There are a great many models that we could fit to our data, and selecting which model to use involves considerable skill and experience. *All models are wrong, but some models are better than others.* Model choice is one of the most frequently ignored of the big issues involved in learning statistics.

In the past, elementary statistics was taught as a series of recipes that you followed without the need for any thought. This caused two big problems. People who were taught this way never realized that model choice is a really big deal ('I'm only trying to do a *t* test'). And they never understood that assumptions need to be checked ('all I need is the *p* value').

Throughout this book you are encouraged to learn the key assumptions. In order of importance, these are

- random sampling
- constant variance
- normal errors
- independent errors
- additive effects

Crucially, because these assumptions are often *not* met with the kinds of data that we encounter in practice, we need to know what to do about it. There are some things that it is much more difficult to do anything about (e.g. non-random sampling) than others (e.g. non-additive effects).

The book also encourages users to understand that in most cases there are literally hundreds of possible models, and that choosing the best model is an essential part of the process of statistical analysis. Which explanatory variables to include in your model, what transformation to apply to each variable, whether to include interaction terms: all of these are key issues that you need to resolve.

The issues are at their simplest with designed manipulative experiments in which there was thorough randomization and good levels of replication. The issues are most difficult with observational studies where there are large numbers of (possibly correlated) explanatory variables, little or no randomization and small numbers of data points. Much of your data is likely to come from the second category.

## Statistical Modelling

The object is to determine the values of the parameters in a specific model that lead to *the best fit of the model to the data*. The data are sacrosanct, and they tell us what actually happened under a given set of circumstances. It is a common mistake to say 'the data were fitted to the model' as if the data were something flexible, and we had a clear picture of the structure of the model. On the contrary, what we are looking for is the minimal adequate model to describe the data. *The model is fitted to data*, not the other way around. The best model is the model that produces the least unexplained variation (the *minimal residual deviance*), subject to the constraint that the parameters in the model should all be statistically significant.

You have to specify the model. It embodies your mechanistic understanding of the factors involved, and of the way that they are related to the response variable. We want the model to be *minimal* because of the principle of parsimony, and *adequate* because there is no point in retaining an inadequate model that does not describe a significant fraction of the variation in the data. It is very important to understand that *there is not one model*; this is one of the common implicit errors involved in traditional regression and ANOVA, where the same models are used, often uncritically, over and over again. In most circumstances, there will be a large number of different, more or less plausible models that might be fitted to any given set of data. Part of the job of data analysis is to determine

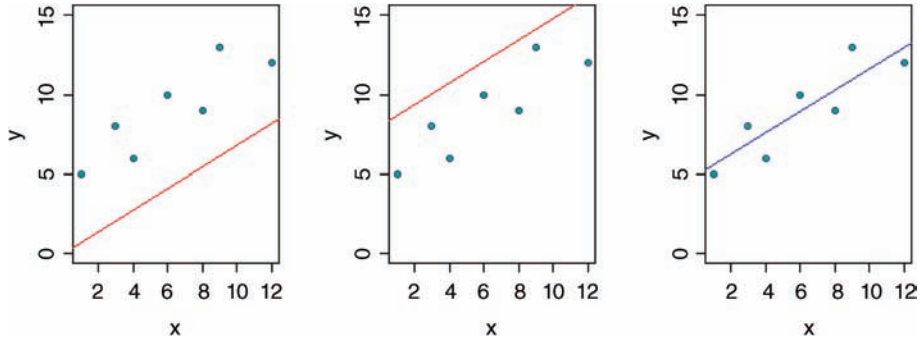
which, if any, of the possible models are adequate, and then, out of the set of adequate models, which is the minimal adequate model. In some cases there may be no single best model and a set of different models may all describe the data equally well (or equally poorly if the variability is great).

### Maximum Likelihood

What, exactly, do we mean when we say that the parameter values should afford the 'best fit of the model to the data'? The convention we adopt is that our techniques should lead to *unbiased, variance minimizing estimators*. We define 'best' in terms of *maximum likelihood*. This notion is likely to be unfamiliar, so it is worth investing some time to get a feel for it. This is how it works:

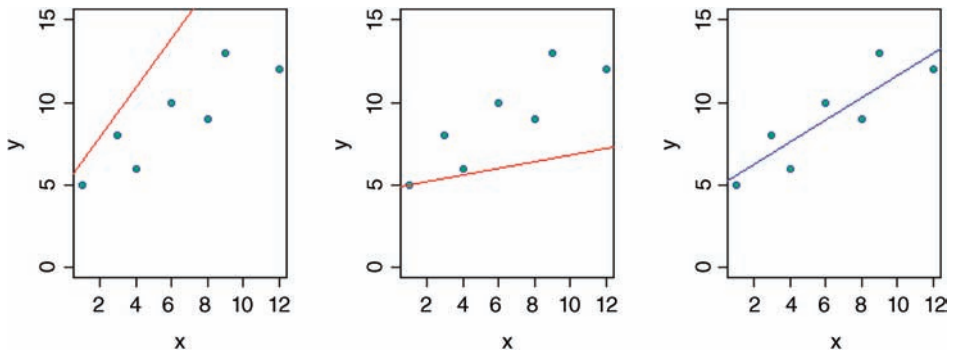
- given the data
- and given our choice of model
- what values of the parameters of that model
- make the observed data most likely?

Let us take a simple example from linear regression where the model we want to fit is $y = a + bx$ and we want the best possible estimates of the two parameters (the intercept $a$ and the slope $b$) from the data in our scatterplot.




If the intercept were 0 (left-hand graph, above), would the data be likely? The answer of course, is no. If the intercept were 8 (centre graph) would the data be likely? Again, the answer is obviously no. The maximum likelihood estimate of the intercept is shown in the right-hand graph (its value turns out to be 4.827). Note that the point at which the graph cuts the $y$ axis is *not* the intercept when (as here) you let R decide where to put the axes.

We could have a similar debate about the slope. Suppose we knew that the intercept was 4.827, then would the data be likely if the graph had a slope of 1.5 (left-hand graph, below)?




The answer, of course, is no. What about a slope of 0.2 (centre graph)? Again, the data are not at all likely if the graph has such a gentle slope. The maximum likelihood of the data given the model is obtained with a slope of 0.679 (right-hand graph).

This is not how the procedure is carried out in practice, but it makes the point that we judge the model on the basis *how likely the data would be if the model were correct*. When we do the analysis in earnest, both parameters are estimated simultaneously.

**Experimental Design**

There are only two key concepts:

- replication
- randomization

You replicate to increase reliability. You randomize to reduce bias. If you replicate thoroughly and randomize properly, you will not go far wrong.

There are a number of other issues whose mastery will increase the likelihood that you analyse your data the right way rather than the wrong way:

- the principle of parsimony
- the power of a statistical test
- controls
- spotting pseudoreplication and knowing what to do about it
- the difference between experimental and observational data (non-orthogonality)

It does not matter very much if you cannot do your own advanced statistical analysis. If your experiment is properly designed, you will often be able to find somebody to help you with the stats. But if your experiment is not properly designed, or not thoroughly randomized, or lacking adequate controls, then no matter how good you are at stats, some (or possibly even all) of your experimental effort will have been wasted. No amount of high-powered statistical analysis can turn a bad experiment into a good one. R is good, but not that good.

### The Principle of Parsimony (Occam's Razor)

One of the most important themes running through this book concerns model simplification. The principle of parsimony is attributed to the fourteenth-century English nominalist philosopher William of Occam who insisted that, given a set of equally good explanations for a given phenomenon, then *the correct explanation is the simplest explanation*. It is called Occam's razor because he 'shaved' his explanations down to the bare minimum. In statistical modelling, the principle of parsimony means that:

- models should have as few parameters as possible
- linear models should be preferred to non-linear models
- experiments relying on few assumptions should be preferred to those relying on many
- models should be pared down until they are *minimal adequate*
- simple explanations should be preferred to complex explanations

The process of model simplification is an integral part of statistical analysis in R. In general, a variable is retained in the model only *if it causes a significant increase in deviance when it is removed from the current model*. Seek simplicity, then distrust it.

In our zeal for model simplification, we must be careful not to throw the baby out with the bathwater. Einstein made a characteristically subtle modification to Occam's razor. He said: 'A model should be as simple as possible. But no simpler.'

### Observation, Theory and Experiment

There is no doubt that the best way to solve scientific problems is through a thoughtful blend of observation, theory and experiment. In most real situations, however, there are constraints on what can be done, and on the way things can be done, which mean that one or more of the trilogy has to be sacrificed. There are lots of cases, for example, where it is ethically or logistically impossible to carry out manipulative experiments. In these cases it is doubly important to ensure that the statistical analysis leads to conclusions that are as critical and as unambiguous as possible.

### Controls

No controls, no conclusions.

### Replication: It's the *n*s that Justify the Means

The requirement for replication arises because if we do the same thing to different individuals we are likely to get different responses. The causes of this heterogeneity in response are many and varied (genotype, age, sex, condition, history, substrate, microclimate, and so on). The object of replication is to increase the reliability of parameter estimates, and to allow us to quantify the variability that is found within the same treatment. To qualify as replicates, the repeated measurements:

- must be independent
- must not form part of a time series (data collected from the same place on successive occasions are not independent)
- must not be grouped together in one place (aggregating the replicates means that they are not spatially independent)
- must be measured at an appropriate spatial scale
- ideally, one replicate from each treatment ought to be grouped together into a block, and all treatments repeated in many different blocks.
- repeated measures (e.g. from the same individual or the same spatial location) are not replicates (this is probably the commonest cause of pseudoreplication in statistical work)

### How Many Replicates?

The usual answer is 'as many as you can afford'. An alternative answer is 30. A very useful rule of thumb is this: a sample of 30 or more is a big sample, but a sample of less than 30 is a small sample. The rule doesn't always work, of course: 30 would be derisively small as a sample in an opinion poll, for instance. In other circumstances, it might be impossibly expensive to repeat an experiment as many as 30 times. Nevertheless, it is a rule of great practical utility, if only for giving you pause as you design your experiment with 300 replicates that perhaps this might really be a bit over the top. Or when you think you could get away with just five replicates this time.

There are ways of working out the replication necessary for testing a given hypothesis (these are explained below). Sometimes we know little or nothing about the variance of the response variable when we are planning an experiment. Experience is important. So are pilot studies. These should give an indication of the variance between initial units before the experimental treatments are applied, and also of the approximate magnitude of the responses to experimental treatment that are likely to occur. Sometimes it may be necessary to reduce the scope and complexity of the experiment, and to concentrate the inevitably limited resources of manpower and money on obtaining an unambiguous answer to a simpler question. It is immensely irritating to spend three years on a grand experiment, only to find at the end of it that the response is only significant at $p = 0.08$. A reduction in the number of treatments might well have allowed an increase in replication to the point where the same result would have been unambiguously significant.

### Power

The power of a test is the probability of rejecting the null hypothesis when it is false. It has to do with Type II errors: $\beta$ is the probability of accepting the null hypothesis when it is false. In an ideal world, we would obviously make $\beta$ as small as possible. But there is a snag. The smaller we make the probability of committing a Type II error, the greater we make the probability of committing a Type I error, and rejecting the null hypothesis when, in fact, it is correct. A compromise is called for. Most statisticians work with $\alpha = 0.05$ and $\beta = 0.2$. Now the power of a test is defined as $1 - \beta = 0.8$ under the standard assumptions. This is

used to calculate the sample sizes necessary to detect a specified difference when the error variance is known (or can be guessed at).

Let's think about the issues involved with power analysis in the context of a Student's *t*-test to compare two sample means. As explained on p. 91, the test statistic is $t = \text{difference}/(\text{the standard error of the difference})$ and we can rearrange the formula to obtain $n$, the sample size necessary in order that that a given difference, $d$, is statistically significant:

$$n = \frac{2s^2t^2}{d^2}$$

You can see that the larger the variance $s^2$, and the smaller the size of the difference, the bigger the sample we shall need. The value of the test statistic $t$ depends on our decisions about Type I and Type II error rates (conventionally 0.05 and 0.2). For sample sizes of order 30, the $t$ values associated with these probabilities are 1.96 and 0.84 respectively: these add to 2.80, and the square of 2.80 is 7.84. To the nearest whole number, the constants in the numerator evaluate to $2 \times 8 = 16$. So as a good rule of thumb, the sample size you need in each treatment is given by

$$n = \frac{16s^2}{d^2}$$

We simply need to work out 16 times the sample variance (obtained from the literature or from a small pilot experiment) and divide by the square of the difference that we want to be able to detect. So suppose that our current cereal yield is 10 t/ha with a standard deviation of sd = 2.8 t/ha (giving $s^2 = 7.84$) and we want to be able to say that a yield increase (delta) of 2 t/ha is significant at 95% with power = 80%, then we shall need to have $16 \times 7.84/4 = 31.36$ replicates in each treatment. The built in R function

```
power.t.test(delta=2,sd=2.8,power=0.8)
```

also gives $n = 32$ replicates per treatment on rounding-up.

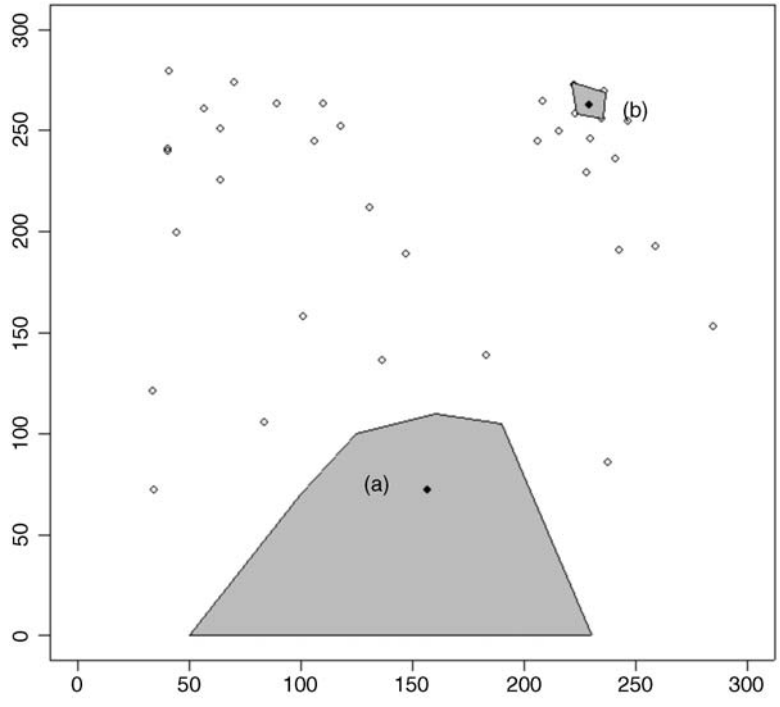
### Randomization

Randomization is something that everybody says they do, but hardly anybody does properly. Take a simple example. How do I select one tree from a forest of trees, on which to measure photosynthetic rates? I want to select the tree at random in order to avoid bias. For instance, I might be tempted to work on a tree that had accessible foliage near to the ground, or a tree that was close to the lab. Or a tree that looked healthy. Or a tree that had nice insect-free leaves. And so on. I leave it to you to list the biases that would be involved in estimating photosynthesis on any of those trees.

One common way of selecting a 'random' tree is to take a map of the forest and select a random pair of coordinates (say 157 m east of the reference point, and 228 m north). Then pace out these coordinates and, having arrived at that particular spot in the forest, select the nearest tree to those coordinates. But is this really a randomly selected tree?

If it *were* randomly selected, then it would have *exactly the same chance of being selected as every other* tree in the forest. Let us think about this. Look at the figure below, which shows a map of the distribution of trees on the ground. Even if they were originally planted out in regular rows, accidents, tree-falls, and heterogeneity in the substrate would soon lead

to an aggregated spatial distribution of trees. Now ask yourself how many different random points would lead to the selection of a given tree. Start with tree (a). This will be selected by any points falling in the large shaded area.




Now consider tree (b). It will only be selected if the random point falls within the tiny area surrounding that tree. Tree (a) has a much greater chance of being selected than tree (b), and so *the nearest tree to a random point is not a randomly selected tree*. In a spatially heterogeneous woodland, isolated trees and trees on the edges of clumps will always have a higher probability of being picked than trees in the centre of clumps.

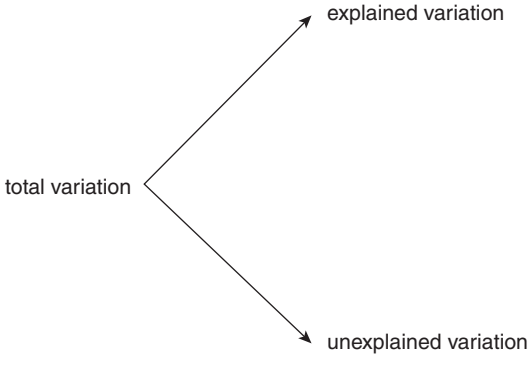
The answer is that to select a tree at random, every single tree in the forest must be numbered (all 24 683 of them, or whatever), and then a random number between 1 and 24 683 must be drawn out of a hat. There is no alternative. Anything less than that is not randomization.

Now ask yourself how often this is done in practice, and you will see what I mean when I say that randomization is a classic example of 'Do as I say, and not do as I do'. As an example of how important proper randomization can be, consider the following experiment that was designed to test the toxicity of five contact insecticides by exposing batches of flour beetles to the chemical on filter papers in Petri dishes. The animals walk about and pick up the poison on their feet. The *Tribolium* culture jar was inverted, flour and all, into a large tray, and beetles were collected as they emerged from the flour. The animals were allocated to the five chemicals in sequence; three replicate Petri dishes were treated with the first chemical, and 10 beetles were placed in each Petri dish. Do you see the source of bias in this procedure?

It is entirely plausible that flour beetles differ in their activity levels (sex differences, differences in body weight, age, etc.). The most active beetles might emerge first from the pile of flour. These beetles all end up in the treatment with the first insecticide. By the time we come to finding beetles for the last replicate of the fifth pesticide, we may be grubbing round in the centre of the pile, looking for the last remaining *Tribolium*. This matters, because the amount of pesticide picked by the beetles up will depend upon their activity levels. The more active the beetles, the more chemical they pick up on their feet, and the more likely they are to die. Thus, the failure to randomize will bias the result in favour of the first insecticide because this treatment received the most active beetles.

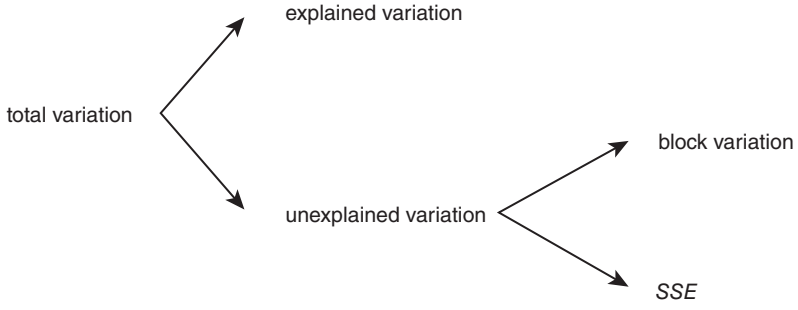
What we should have done is this. If we think that insect activity level is important in our experiment, then we should take this into account at the design stage. We might decide to have three levels of activity: active, average and sluggish. We fill the first five Petri dishes with 10 each of the active insects that emerge first from the pile. The next 50 insects we find go 10-at-a-time into five Petri dishes that are labelled average. Finally, we put last 50 insects to emerge into a set of five Petri dishes labelled sluggish. This procedure has created three *blocks* based on activity levels: we do not know precisely why the insects differed in their activity levels, but we think it might be important. Activity level is called a *random effect*: it is a factor with three levels. Next comes the randomization. We put the names of the five insecticides into a hat, shuffle them up, and draw them out one-at-a-time at random. The first Petri dish containing active beetles receives the insecticide that is first out of the hat, and so on until all five active Petri dishes have been allocated their own different pesticide. Then the five labels go back in the hat and are reshuffled. The procedure is repeated to allocate insecticide treatment at random to the five average activity Petri dishes. Finally, we put the labels back in the hat and draw the insecticide treatment for the five Petri dishes containing sluggish insects.

But why go to all this trouble? The answer is very important, and you should read it again and again until you understand it. The insects differ and the insecticides differ. But the Petri dishes may differ, too, especially if we store them in slightly different circumstances (e.g. near to the door of the controlled temperature cabinet or away at the back of the cabinet). The point is that there will be a total amount of variation in time to death across all the insects in the whole experiment (all $3 \times 5 \times 10 = 150$ of them). We want to partition this variation into that which can be explained by differences between the insecticides and that which cannot.



If the amount of variation explained by differences between the insecticide treatments is large, then we conclude that the insecticides are significantly different from one another in their effects on mean age at death. We make this judgement on the basis of a comparison between the explained variation *SSA* and the unexplained variation *SSE*. If the unexplained variation is large, it is going to be very difficult to conclude anything about our *fixed effect* (insecticide in this case).

The great advantage of blocking is that it reduces the size of the unexplained variation. In our example, if activity level had a big effect on age at death (block variation), then the unexplained variation *SSE* would be much smaller than would have been the case if we had ignored activity and the significance of our fixed effect will be correspondingly higher:




The idea of good experimental design is to make *SSE* as small as possible, and blocking is the most effective way to bring this about.

R is very useful during the randomization stage because it has a function called `sample` which can shuffle the factor levels into a random sequence. Put the names of the five insecticides into a vector like this:

```
treatments <- c("aloprin","vitex","formixin","panto","allclear")
```

Then use `sample` to shuffle them for the active insects in dishes 1 to 5:

```
sample(treatments)
```

```
[1] "formixin" "panto"  "vitex"  "aloprin" "allclear"
```

then for the insects with average activity levels in dishes 6 to 10:

```
sample(treatments)
```

```
[1] "formixin" "allclear" "aloprin" "panto"  "vitex"
```

then finally for the sluggish ones in dishes 11 to 15:

```
sample(treatments)
```

```
[1] "panto"  "aloprin" "allclear" "vitex"  "formixin"
```

The recent trend towards 'haphazard' sampling is a cop-out. What it means is that 'I admit that I didn't randomize, but you have to take my word for it that this did not introduce any important biases'. You can draw your own conclusions.

### Strong Inference

One of the most powerful means available to demonstrate the accuracy of an idea is an experimental confirmation of a prediction made by a carefully formulated hypothesis. There are two essential steps to the protocol of *strong inference* (Platt, 1964):

- formulate a clear hypothesis
- devise an acceptable test

Neither one is much good without the other. For example, the hypothesis should not lead to predictions that are likely to occur by other extrinsic means. Similarly, the test should demonstrate unequivocally whether the hypothesis is true or false.

A great many scientific experiments appear to be carried out with no particular hypothesis in mind at all, but simply to see what happens. While this approach may be commendable in the early stages of a study, such experiments tend to be weak as an end in themselves, because there will be such a large number of equally plausible explanations for the results. Without contemplation there will be no testable predictions; without testable predictions there will be no experimental ingenuity; without experimental ingenuity there is likely to be inadequate control; in short, equivocal interpretation. The results could be due to myriad plausible causes. Nature has no stake in being understood by scientists. We need to work at it. Without replication, randomization and good controls we shall make little progress.

### Weak Inference

The phrase 'weak inference' is used (often disparagingly) to describe the interpretation of observational studies and the analysis of so-called 'natural experiments'. It is silly to be disparaging about these data, because they are often the only data that we have. The aim of good statistical analysis is to obtain the maximum information from a given set of data, *bearing the limitations of the data firmly in mind*.

Natural experiments arise when an event (often assumed to be an unusual event, but frequently without much justification of what constitutes unusualness) occurs that is like an experimental treatment (a hurricane blows down half of a forest block; a landslide creates a bare substrate; a stock market crash produces lots of suddenly poor people, etc.). 'The requirement of adequate knowledge of initial conditions has important implications for the validity of many natural experiments. Inasmuch as the "experiments" are recognized only when they are completed, or in progress at the earliest, it is impossible to be certain of the conditions that existed before such an "experiment" began. It then becomes necessary to make assumptions about these conditions, and any conclusions reached on the basis of natural experiments are thereby weakened to the point of being hypotheses, and they should be stated as such' (Hairston, 1989).

### How Long to Go On?

Ideally, the duration of an experiment should be determined in advance, lest one falls prey to one of the twin temptations: