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Dinamica dei gas rarefatti

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FONDAZIONE CIME ROBERTO CONTI

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Dinamica dei gas rarefatti

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DINAMICA DEI GAS RAREFATTI

Premessa al Ciclo sulla "Dinamica dei Gas Rarefatti"

di

CARLO FERRARI

Desidero porgere innanzi tutto il più cordiale benvenuto ad ognuno dei presenti, ed esprimere un particolare ringraziamento ai cari ed illustri Colleghi che nella Sessione C.I.M.E. che oggi si inizia, si sobbarcano alla maggiore fatica: quella di dare in un ciclo di conferenze relativamente breve la più larga, e nello stesso tempo la più completa possibile trattazione dei problemi che si riferiscono alla dinamica dei gas rarefatti; e senza dubbio, grazie alla loro ben nota capacità e altissima competenza sull'argomento, allo studio del quale tanti contributi originali essi hanno dato, essi riusciranno nell'intento, che pure è difficile anche se l'assemblea degli ascoltatori è per molti effetti eccezionale.

I problemi della dinamica dei gas rarefatti hanno da tempo attirato l'attenzione di Matematici, di Fisici, di Chimici: basti ricordare i nomi di Knudsen, Smoluchowsky, Enkscog, Clausing ; i tentativi fatti per ottenere soluzioni approssimate dell'equazione di Boltzmann; gli studi teorici e sperimentali sull'adsorbimento fisico e chimico di gas da parte di superfici solide ; le ricerche sui processi di diffusione attraverso a materiali porosi per la separazione degli isotopi.

Ma le applicazioni recenti della meccanica dei fluidi hanno proposto nuovi problemi o hanno riproposto problemi vecchi sotto veste nuova ; l'uso di satelliti artificiali e di razzi spaziali per investigare la struttura e le proprietà della ionosfera e del mezzo interplanetario hanno intensificato l'interesse nello studio degli effetti, che si producono in vicinanza di un corpo che si muove in un mezzo molto rarefatto. D'altra parte questo mezzo rarefatto non è un aggregato di molecole e di atomi neutri, ma è un plasma, ossia un aggregato di molecole, atomi, ioni ed elettroni. In effetto nella parte inferiore della ionosfera, ad un'altezza di 200 Km. dalla superficie terrestre, il numero di densità delle particelle neutre è da 2 a 5×10^{10} particelle per cm^3., con un percorso libero medio di 8×10 cm.; mentre il numero di densità degli elettroni o degli ioni è da 3 a $50 \times 10^4/cm^3$. con un percorso libero medio di 9×10^3 cm. A 300 Km. di altezza i numeri di densità delle particelle neutre e di quelle cariche diventano rispettivamente 3×10^9 e $1 \div 2 \times 10^6$, e i loro percorsi liberi medi 10^5 cm.; 7×10^3 cm. Infine a 3000 Km. di altezza si può presumere che le particelle neutre siano presenti in numero di uno per cm^3, mentre le particelle elettrizzate sono $7 \times 10^3/cm^3$; i percorsi liberi medi sono rispettivamente 2×10^{14} e 3×10^6 cm.

Con i gradi di rarefazione del mezzo quali risultano dai numeri ora detti, gli usuali metodi dell'aerodinamica non possono essere applicati per descrivere i fenomèni che sono prodotti nel mezzo stesso dal moto di un corpo; è necessario applicare la teoria cinetica. Ora, le particelle neutre, molecole o atomi, interagiscono solo colla superficie dell'ostacolo; le particelle cariche invece, non soltanto presentano questo tipo di interazione, ma sono anche influenzate nel loro moto dai campi elettrico e magnetico, ed il campo elettrico a sua volta è prodotto e dalle cariche spaziali, che si producono nel plasma per effetto della differenza di concentrazione degli ioni e degli elettroni causata dalla presenza del corpo, e dalla carica che questo assume. I problemi che sono connessi allo studio di queste interazioni possono variare notevolmente: nella parte inferiore della ionosfera, la velocità del corpo $\underline{v}_o$ è generalmente molto maggiore della velocità termica degli ioni $\underline{v}_i$, e molto più piccola di quella $\underline{v}_e$ degli elettroni. Così, all'altezza di 200 Km.

dette velocità sono rispettivamente dell'ordine : $v_o \propto 10^6$ cm/sec ; $v_i = \propto 10^4$ cm/sec. i $v_e = \propto 10^7$ cm/sec. e pertanto il moto del corpo e supersonico rispetto agli ioni, subsonico rispetto agli elettroni. Ma passando dagli strati inferiori a quelli superiori della ionosfera i parametri caratteristici del plasma nel quale il corpo si muove cambiano sostanzialmente : la velocità termica $\underline{v}_i$ degli ioni aumenta , diventando dell'ordine di grandezza 10^5 cm/sec. (all'altezza di 700 Km.), mentre la velocità del corpo $\underline{v}_o$ diminuisce, di guisa che $\underline{v}_o$ e $\underline{v}_i$ diventano uguali, od anche può verificarsi la condizione inversa a quella prima indicata, ed il moto è subsonico rispetto ad entrambe le particelle cariche. D'altra parte, la lunghezza di Debye, che nella ionosfera è alquanto più piccola della lunghezza che caratterizza la dimensione trasversale dell'ostacolo (precisamente, $1 \div 4$ cm. contro 1 metro), diventa nello spazio interplanetario dello stesso ordine di detta dimensione. Al variare delle caratteristiche del plasma variano, ovviamente, gli effetti dovuti al moto del corpo ; in particolare, variano il carattere del "riempimento" e le dimensioni della regione di "rarefazione" (rispetto all'ambiente indisturbato), dietro all'ostacolo, che superano notevolmente quelle dell'ostacolo stesso, e che presentano tanta importanza nello studio delle perturbazioni della propagazione delle onde elettromagnetiche.

Non ho voluto, e non ho potuto, che fare un cenno a problemi di grande interesse attuale, che la dinamica dei gas rarefatti presenta, e che già da soli giustificherebbero l'esistenza di una speciale Sessione del C.I.M.E. dedicata a questa importante branca della Fisica-Matematica. I fondamenti matematici e fisici per lo studio di questi e di altri problemi di uguale interesse, insieme colla trattazione di alcune tipiche applicazioni dei metodi e delle equazioni fondamentali della dinamica dei gas rarefatti verrano esposti nelle conferenze dei professori Estermann, Kampé de Fériet, Krywoblocki, Lunc, ai quali come già ho detto in principio, molto deve il progresso delle ricerche in questo campo. Conferenze di Seminario sopra problemi particolari, relativi alla interazione tra superfici solide e flusso di molecole libero, e al problema di Rayleygh in magnetogasdinamica, saranno tenute dal professor Nocilla e dai dottori Tironi e Sergianotto, mentre altre conferenze sull'onda d'urto in gas rerefatto in magnetogasdinamica , e sulla struttura dell'alta atmosfera saranno fatte dai profèssori Agostinelli e Graffi. Dai nomi che ora ho detto appare l'internazionalità dei Conferenzieri ; d'altra parte anche gli altri Studiosi qui presenti sono venuti da ogni parte d'Italia e da diverse Nazioni, così che questa è certo una assemblea internazionale altamente qualificata, in cui lo spirito di collaborazione, come sempre avviene nelle riunioni di persone accomunate dall'amore alla ricerca scientifica, è pieno e sincero. Ma qui a Varenna c'è una particolare atmosfera, l'atmosfera di "Villa Monastero" che rende i contatti personali particolarmente cordiali e fecondi, come hanno dimostrato i vari Corsi che qui si sono succeduti. Auguro che questa atmosfera sia ugualmente salutare per lo sviluppo delle ricerche, di cui ora qui ci stiamo occupando, e delle relazioni e collaborazioni personali con amici vecchi e nuovi, e con questo augurio apro la presente Sessione del C.I.M.E.

CENTRO INTERNAZIONALE MATEMATICO ESTIVO

(C. I. M. E.)

M. Z. v. KRZYWOBLOCKI

SOME MATHEMATICAL ASPECTS OF RAREFIED GASDYNAMICS AS APPLIED TO HYPERSONICS, REENTRY AND MAGNETO-GAS-DYNAMICS.

SOME MATHEMATICAL ASPECTS OF RAREFIED GASDYNAMICS AS APPLIED TO HYPERSONICS, REENTRY AND MAGNETO-GAS-DYNAMICS

by M. Z. v. Krzywoblocki
(Michigan State University)

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M. Z. v. Krzywoblocki

SUMMARY

The author presents briefly the fundamental equations of the free molecule flow regime and of the remaining regions of the hypersonic flow. He discusses the recent developments in three techniques, possibly applicable to the hypersonic flow : integral operators, reduction of independent variables and topological technique.

A brief presentation of the fundamentals of the relativistic energodynamics and some concluding remarks close the paper.

INTRODUCTION

There exists no precise definition of the boundaries of the regime known under the name of hypersonics. On one side it includes the reentry phenomena which are inseparably connected with all the domains of the classical aero- and fluid-dynamics, involving all of the classical regions of the sub-, trans-, and super-sonic nature. On the other side it involves the relativistic phenomena with the velocity of light being another barrier, the light-barrier, having a some sort of analogy to the old sonic barrier. Practically, the hypersonics is interested in the sub-light regime, leaving the super-light region still in the sphere of speculations of the theoretical physics. Hypersonics applies all the possible tools, ever invented, developed and worked out by the mechanics of continuous media, kinetic theory of gases, Newtonian free molecule technique and finally the classical special theory of relativity. This refers to all kinds of gaseous media, i. e., ideal, perfect, real, ionized, etc. There is no limit in that respect from any point of view. Concerning the problem of solving the differential and integral equations occurring in the fields in question, there are used all of the possible techniques and methods known in the theory of partial and ordinary differential as well as of integro-differential equations. One can mention here the classical techniques, special functions, algebraic, integral operator, topological techniques, reduction of the number of independent variables, etc. Each of these methods possesses certain advantages and disadvantages.

The present work is concerned with certain aspects of the enormously large field of hypersonics, namely with the discussion of some recent developments in a few techniques of remodelling the equations, governing the flow of a gas in that region. Obviously, not only the state of a neutral gas but as well that of an ionized gas (electro-magneto-hydrodynamics or plasma-dynamics) should be considered.

In general, the classical techniques are not strong enough to attack the nonlinear systems. Integral operator technique discussed in the present work, involves a hodograph transformation with all its enormously complicated formalism of returning back to the physical plane, difficulties with boundary conditions (unknown) in the hodograph plane, etc.

The technique of the reduction of independent variables is actually based upon the elementary fundamentals of the theory of invariant groups.

As such one, it does not take account of the boundary conditions.

The topological techniques furnish usually in a very simple manner the existence proof. But concerning the formal solution at a point they seem to require a lot of formalism involving necessarily numerical procedures usually with the use of high-speed computing machines.

Actually, all the techniques, discussed in the present work require necessarily the application of numerical solutions and the use of the high speed computing machines. There is briefly discussed the most recent tendency in calculating the reentry phenomena of blunt bodies (Apollo's shape) by simply programming directly the equations of motion for the high speed computing machines without any whatsoever remodelling them from their original forms.

Finally, tha last part of the work is concerned with the approach to the hypersonic flow regime from the light-barrier point of view. This is accomplished by discussing the fundamentals of the relativistic energo-dynamics. It involves the invariance of the total energy of the system in question filled out by a continuous (matter-full) medium under the transformation group of coordinates. The assumption is that the matter (i. e., a certain level of energy) can be transformed into the energy of the light (electro-magnetic-matterless energy). This approach could be of some value in motion of matter-full particles in the range of velocities approaching the velocity of light. Some concluding remarks close the work.

1. Free Molecule Flow Technique And The Fundamental Systems Of Equations

1. 1. Free Molecule Flow

In this chapter we shall briefly discuss the fundamental aspects of the free molecule flow (no collision between particles of the gas only the impact of the gas-molecules on the surface of the moving body), reflecting them from more than one point of view. Next we shall collect some fundamental systems of equations used to describe hypersonic flow in its various aspects.

If ξ', η', ζ' are the components of velocity of a molecule in the directions x', y', z' of a coordinate system in which the macroscopic velocity of the gas is zero, then since the velocity distribution is not modified by the impact with the body because of absence of collision between molecules in the free stream and re-emitted molecules from the surface of the body, the distribution is Maxwellian, or

$$N_{\xi'\eta'\zeta'} = N(H/\pi)^{3/2} \exp\left\{-h(\xi'^2 + \eta'^2 + \zeta'^2)\right\}. \tag{1.1.1}$$

$N_{\xi'\eta'\zeta'}$ is the number of molecules per unit volume in the range of velocities ξ' to $\xi' + d\xi'$, η' to $\eta' + d\eta'$, and ζ' to $\zeta' + d\zeta'$ divided by $d\xi' d\eta' d\zeta'$. N is total of molecules per unit volume. h is related to the most probable velocity c_i of the molecules in the free stream by the expression

$$h = 1/c_i^2 \quad , \tag{1.1.2}$$

where $c_i^2 = 2RT = 2(p/\rho)$.

If the observer moves with a velocity e_1U, e_2U, e_3U in the directions x', y', z', then in the relative coordinate system x, y, z where the observer is considered at rest, the velocities ξ, η, ζ of the molecule in the directions x, y, z are

(1. 1. 3) $\xi = \xi' - e_1 U, \quad \eta = \eta' - e_2 U, \quad \zeta = \zeta' - e_3 U .$

Therefore, in the new coordinate system, the number of molecules having velocity component between ξ, η, ζ and $\xi + d\xi$, $\eta + d\eta$, $\zeta + d\zeta$ is

(1. 1. 4) $N_{\xi\eta\zeta} = N(h/\pi)^{3/2} \exp\left\{-h(\xi + e_1 U)^2 + (\eta + e_2 U)^2 + (\zeta + e_3 U)^2\right.$

Suppose there is a surface dS whose normal is the x-axis. During a unit second the molecules, having velocity components between ξ, η, ζ and $\xi + d\xi$, $\eta + d\eta$, $\zeta + d\zeta$, and striking the surface dS, will be contained at a given moment in the cylinder with dS as base and the length $\sqrt{\xi^2 + \eta^2 + \zeta^2}$ in the direction ξ, η, ζ.

The volume of the cylinder is equal to the area of the base dS multiplied by the heights $(-\xi)$. The number of this kind of molecules is then $N_{\xi\eta\zeta}\,(-\xi)dS$. The number of molecules between ξ, η, ζ and $\xi + d\xi$, $\eta + d\eta$, $\zeta + d\zeta$ that will strike a unit area with x-axis as normal is then $-\xi N_{\xi\eta\zeta}\, d\xi\, d\eta\, d\zeta$. The total number n of molecules striking this unit area is

(1. 1. 5) $n = -N\left(\frac{h}{\pi}\right)^{3/2} \int_{-\infty}^{0} d\xi \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} \xi\, d\xi \,\exp\left\{-h\left[(\xi + e_1 U)^2 + (\eta + e^2 U)^2 + (\zeta + e_3 U)^2\right]\right\}$

the result being

(1. 1. 6) $n = N\left\{\left[2(\pi h)^{1/2}\right]^{-1} \exp\left[-h(e_1 U)^2\right] + 2^{-1} e_1 U\left[1 + \operatorname{erf}(e_1 U\sqrt{h})\right]\right\}$

where erf(t) is the error function defined as

(1. 1. 7) $\operatorname{erf}(t) = 2(\pi)^{-1/2} \int_0^t \exp(-s^2)ds .$

Hence, the number of molecules per second striking a unit area of the plate inclined at angle θ to the stream with velocity U is

(1. 1. 8) $$n = \frac{N}{2}\left\{(\pi h)^{-1/2}\exp\left[-h(U\sin\theta)^2\right] + U\sin\theta\left[1+\mathrm{erf}(Uh^{1/2}\sin\theta)\right]\right\}$$

The mass m_i of the stream per second striking a unit area of the plate is

(1. 1. 9) $$m_i = \frac{c_i\rho}{2\pi^{1/2}}\left\{\exp\left[-(Uc_i^{-1}\sin\theta)^2\right] + \pi^{1/2}Uc_i^{-1}\sin\theta\left[1+\mathrm{erf}(Uc_i^{-1}\sin\theta)\right]\right\}$$

The component of velocity of the molecule in a direction, having directional cosines e_1', e_2', e_3' with the axes is $e_1'\xi + e_2'\eta + e_3'\zeta$. If this molecule is absorbed by the surface after striking it, the corresponding momentum will be transferred to the surface. For the surface with x-axis as normal considered as above, the total momentum $M_{e_1'e_2'e_3'}$ per second per unit area is

(1. 1. 10) $$M_{e_1'e_2'e_3'} = -\rho(h\pi^{-1})^{3/2}\int_{-\infty}^{0}d\xi\int_{-\infty}^{\infty}d\eta\int_{-\infty}^{\infty}\xi(e_1'\xi + e_2'\eta + e_3'\zeta).$$
$$.\exp\left\{-h\left[(\xi+e_1U)^2+(\eta+e_2U)^2+(\zeta+e_3U)^2\right]\right\}d\zeta\;,$$

or

(1. 1. 11) $$M_{e_1'e_2'e_3'} = -\frac{\rho}{2}U^2\left\{(\pi h)^{-1/2}U^{-1}(e_1e_1'+e_2e_2'+e_3e_3')\exp\left[-h(e_1U)^2\right] + \right.$$
$$\left.+\left[e_1'(2hU^2)^{-1}+e_1(e_1e_1'+e_2e_2'+e_3e_3')\right].\left[1+\mathrm{erf}(Ue_1h^{1/2})\right]\right\}.$$

The pressure p_i due to impact of molecules on a plate inclined at an angle θ to the stream of velocity U is calculated by using

(1. 1. 12) $$e_1 = \sin\theta\;,\; e_2 = \cos\theta\;,\; e_3 = 0;\; e_1' = -1,\; e_2' = 0,\; e_3' = 0\;,$$

and the impact pressure p_i is

(1. 1. 13) $$\frac{p_i}{\frac{1}{2}\rho U^2} = \pi^{-1/2}\sin\theta\;(c_iU^{-1})\exp\left[-(Uc_i^{-1}\sin\theta)^2\right] +$$
$$+\left[2^{-1}(c_iU^{-1})^2+\sin^2\theta\right].\left[1+\mathrm{erf}(Uc_i^{-1}\sin\theta)\right].$$

To calculate the shearing stress τ_i due to impact of molecules on the plate, the direction cosines are

(1. 1. 14) $$e_1 = \sin\theta\ ,\ e_2 = \cos\theta\ ,\ e_3 = 0;\ e'_1 = 0,\ e'_2 = -1,\ e'_3 = 0\ .$$

By substituting Eq. (1. 1. 14) into Eq. (1. 1. 11), we obtain

(1. 1. 15) $$\frac{\tau_i}{\frac{1}{2}\rho U^2} = \cos\theta\ .\ c_i U^{-1} \exp\left[-(Uc_i^{-1}\sin\theta\ )^2\right] + \sin\theta\ \cos\theta\ \left[1 + \operatorname{erf}(Uc_i^{-1}\sin\theta\ )\right].$$

If the molecules reflect specularly from the surface, then the pressure and the shearing stress due to re-emission are

(1. 1. 16) $$p_r = p_i\ \ ;\ \ \tau_r = -\tau_i\ \ .$$

On the other hand, if molecules re-emit diffusively, $\tau_r = 0$, and the pressure p_r due to diffuse-reemission, is given by

(1. 1. 17) $$\frac{p_r}{\frac{1}{2}\rho U^2} = 2^{-1}\pi^{1/2} c_r U^{-1}\left\{c_i U^{-1}\ \pi^{-1/2}\exp\left[-(Uc_i^{-1}\sin\theta\ )^2\right] + \sin\theta\left[1 + \operatorname{erf}(Uc_i^{-1}\sin\theta\ )\right]\right\}\ ,$$

where c_r is the most probable velocity of the molecule in the termal equilibrium at a temperature T_r of the reemitted gas, having the relation

(1. 1. 18) $$c_r^2 = 2RT_r\ \ .$$

The above presentation is actually a part of the work by Tsien [72] .

1. 2. Kinetic Theory of Gases Approach.

The approach to the free molecule flow from the kinetic theory of gases was proposed by Heineman [37] and Keller [42] .

One can find the momentum per second imparted to dS' by the impinging only :

$$-m\int_{-\infty}^{0} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw.\, u(u\cos\theta + v\sin\theta). \tag{1.2.1}$$
$$. f(u+V\cos\theta, v+V\sin\theta, w, xyz)dS'.$$

Here the symbol f denotes the distribution function of the gas. Then total momentum per second D_i for the impinging molecules is

$$D_i = -m\Big\{\int dS' \int_{-\infty}^{0} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u(u\cos\theta + v\sin\theta).$$
$$. f(u+V\cos\theta, v+V\sin\theta, w, xyz) - \tag{1.2.2}$$
$$- \int dS'' \int_{0}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u(u\cos\theta + v\sin\theta).$$
$$. f(u+V\cos\theta, v+V\sin\theta, w, xyz)\Big\}.$$

The total momentum per second exerted by the reflected molecules is

$$D_r^{(s)} = -m\Big\{\int dS' \int_{0}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u(u\cos\theta + v\sin\theta).$$
$$. f(u-V\cos\theta, v+V\sin\theta, w, xyz) - \tag{1.2.3}$$
$$- \int dS'' \int_{-\infty}^{0} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u(u\cos\theta + v\sin\theta).$$
$$. f(u-V\cos\theta, v+V\sin\theta, w, xyz)\Big\},$$

for specular reflection, and

$$D_r^{(d)} = -m\Big\{\int dS' \int_{0}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, A_f u(u\cos\theta + v\sin\theta)\exp(-h_r c^2) - \tag{1.2.4}$$
$$- \int dS'' \int_{-\infty}^{0} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, A_b u(u\cos\theta + v\sin\theta)\exp(-h_r c^2)\Big\},$$

for diffuse reflection as the distribution of reflected molecules is of the form $A\exp(-h_r c^2)$; here $h_r = m(2KT_r)^{-1}$ where T_r is the absolute temperature of the reflected stream, $c^2 = u^2+v^2+w^2$. A_f and A_b can be determined from the conservation of energy and number at the surface. A_f and A_b are given by

$$A_f \int_0^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u \exp(-h_r c^2) = - \int_{-\infty}^{0} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw \,. \tag{1.2.5}$$
$$. \, u f(u+V\cos\theta \,,\; v+V\sin\theta \,, w, xyz),$$

$$A_b \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw\, u \exp(-h_r c^2) = - \int_0^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dw \,. \tag{1.2.6}$$
$$. \, u f(u+V\cos\theta \,,\; v+V\sin\theta \,, w, xyz).$$

In the first approximation we neglect collisions between the molecules. At infinity the function f is Maxwellian, i. e.,

$$f = f^{(M)} = N(h\,\pi^{-1})^{3/2} \exp\left\{ -h\left[(u+V\cos\theta)^2 + (v+V\sin\theta)^2 + w^2\right]\right\}. \tag{1.2.7}$$

Here N is the number density for the monatomic molecules and at the boundary, $f = f^{(M)} + f^{(R)}$ where $f^{(R)}$ is the distribution function of the reflected stream. Eqs(1. 2. 5) and (1. 2. 6) now take the form

$$A_f = N\,\pi^{-3/2} h_i^{1/2} h_r^2 \Big\{ (\pi h_i^{-1})^{1/2} V\cos\theta + h_i^{-1}\exp(-h_i V^2\cos^2\theta) + \tag{1.2.8}$$
$$+ 2V\cos\theta \int_0^{V\cos\theta} \exp(-h_i u^2) du \Big\}$$

$$A_b = N\,\pi^{-3/2} h_i^{1/2} h_r^2 \Big\{ -(\pi h_i^{-1})^{1/2} V\cos\theta + h_i^{-1}\exp(-h_i V^2\cos^2\theta) + \tag{1.2.9}$$
$$+ 2V\cos\theta \int_0^{V\cos\theta} \exp(-h_i u^2) du \Big\} \,,$$

where $h_i = m(2KT_i)^{-1}$, T_i being the temperature of the impinging stream. Heineman furnishes the drag coefficients for a plate, sphere, right circular cone and prolate ellipsoid. We present here the drag coefficient of a plate :

Specular reflection :

$$C_D = 8(2\pi\gamma)^{-1/2} M^{-2} \Big\{ (2/\gamma)^{1/2} (1+\gamma M^2\cos^2\theta)\, \mathrm{erf}\big((\gamma/2)^{1/2} M\cos\theta\big) + \tag{1.2.10}$$
$$+ M\cos\theta \, \exp(-(\gamma/2) M^2\cos^2\theta) \Big\} .$$

Diffuse reflection :

$$C_D = 4(2\pi\gamma)^{-1/2}(M_i^2\cos\theta)^{-1}\Big\{(2/\gamma)^{1/2}(1+\gamma M_i^2)\cos\theta\,\mathrm{erf}\big[(\gamma/2)^{1/2}M_i\cos\theta\big]$$

(1. 2. 11)

$$+ M_i\exp\big[-(\gamma/2)M_i^2\cos^2\theta\big]\Big\} + (2\pi\gamma^{-1})^{1/2}M_r^{-1}\cos\theta \;,$$

where M_i and M_r are respectively equal to $V/v_s^{(i)}$ and $V/v_s^{(r)}$, $v_s^{(i)}$ and $v_s^{(r)}$ being the sound velocities in the impinging and reflected streams.

Keller employs the Jaffé's method [41] with more general type of boundary condition. Let s denotes arc length along a trajectory, then the Boltzmann equation may be written :

(1. 2. 12) $$df/ds = a\lambda^{-1}J(f,f) \;, \qquad \lambda = a^3/N\sigma^2 \;,$$

where a is a typical macroscopic dimension; λ is (when divided by π) the mean free path in a gas of spherical molecules of radius σ ; N = number of particles $/a^3$;

(1. 2. 13) $$J(f,f) = \iiint \big[f(c')f(c_1') - f(c)f(c_1)\big] |c-c_1| \, b\,db\,d\varepsilon\, dc_1 \;.$$

We now assume that f can be represented by a convergent power series in :

(1. 2. 14) $$f = f_o + a\lambda^{-1}f_1 + (a\lambda^{-1})^2 f_2 + \dots \;.$$

If this solution is inserted into Eq. (1. 2. 12) and coefficients of like powers of $a\lambda^{-1}$ are equated, the following infinite set of equations is obtained :

(1. 2. 15)

$$df_o/ds = 0 \;, \qquad df_1/ds = J(f_o, f_o) \;,$$

$$df_2/ds = J(f_o, f_1) + J(f_1, f_o) = 2J(f_o, f_1), \dots$$

When ther external forces F_1, F_2, F_3, are constants, the trajectories may be calculated explicitly, and one finds that the general solution for f_o is :

(1. 2. 16) $$f_o = h(x-ut+F_1t^2/2,\ y-vt+F_2t^2/2,\ z-wt+F_3t^2/2,\ u-F_1t,\ v-F_2t,\ w-F_3t) \;,$$

where h is an arbitrary function.

Keller assumes that for every gas molecule which strikes the surface element dS of a solid or liquid surface during the time dt with velocity (relative to that of dS) between c' and $c' + dc'$ there is a probability $p(c, c')\, dc\, dc'$ that a molecule with velocity between c and $c + dc$ will leave the element dS during the interval dt. Further, he assumes that there is a probability $g(c)\, dc\, dS\, dt$ that a particle with velocity between c and $c + dc$ will spontaneously leave the element dS during the time interval dt , and finds the relation which should be satisfied by $f(c)$ at a solid or liquid surface :

$$(1.2.17) \qquad -f(c)c.n = \int_{c'.n > 0} p(c, c')f(c')c'.n\,dc' + g(c) \quad \text{for} \quad c.n < 0$$

where n is a unit normal to dS pointing out of the gas, and $p(c, c')$ is given by :

(i) Specular reflection :

$$(1.2.18) \qquad p_s(c, c') = \begin{cases} \infty & \text{if } c = c' - 2(c'.n)n \\ 0 & \text{if } c \neq c' - 2(c'.n)n \end{cases} , \quad \int_{c.n < 0} p_s(c, c')dc = 1 \, ;$$

(ii) Diffuse reflection :

$$(1.2.19) \qquad \int_{c.n < 0} p_0(c, c')dc = 1 \, ;$$

$$p_0(c, c') = (2\pi)^{-1} m^2 (KT)^{-2} (-c.n) \exp\left[-mc^2 (2KT)^{-1}\right]$$

where temperature T is given by $T = T_g + \alpha(T_s - T_g)$, T_g = temperature of incident molecules, T_s = temperature of the surface, α = coefficient of accomodation, $0 < \alpha < 1$.

(iii) Part specular and part diffuse :

$$(1.2.20) \qquad p(c, c') = f_r p_D + (1 - f_r) p_S \, ; \quad \int_{c.n < 0} p(c, c')dc = 1 \, .$$

The theoretical determination of $p(c, c')$ and $g(c)$ can be based on the quantum mechanical investigation of the interaction of molecules with a solid surface (see [63]).

The drag force and the torque (with respect to a given origin) exerted by a gas on a body in the gas are given by :

(1. 2. 21) $$D = m \int_S \int_c cfc.\, dSdc \ ,$$

(1. 2. 22) $$T = m \int_S \int_c c \times rfc.\, dSdc \ .$$

1. 3. Equations Based On The Theory Of Continuous Media.

The equations of this kind used in hypersonics in any region, mentioned above (sub-, transon, super-sonic), are the well-known equations of Euler and Navier-Stokes. In the field of hypersonics there are usually introduced the following improvements :

(i) the specific heats c_p , c_v , and their ratio, γ must be properly adjusted to the physical nature of the gas (not necessarily the air in the zero-level consistency); since in the mechanics of continuous media these parameters cannot be evaluated from the fundamental concepts, they must be furnished by the experimental physics;

(ii) similarly, the both coefficient of viscosity, their relation and the coefficient of heat conduction resulting from the transport phenomena must be furnished by the experimental physics.

Since the above equations are so well-known, we do not need to present them in the present work.

1. 4. Macroscopic Equations Based Upon The Kinetic Theory Of Gases.

These equations were derived in the past and actually present some average description of the medium in question, which supposedly consists of discrete particles subject to the collision phenomena.

Since these are macroscopic equations we can deal with them in a way similar to that applied to equations based on the mechanics of continuous media.

We briefly quote the following system of equations from [70] :

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{1.4.1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial u_j} + \rho^{-1} \frac{\partial P_{ij}}{\partial x_j} = 0 \tag{1.4.2}$$

$$\frac{\partial p}{\partial t} + \frac{\partial (pu_i)}{\partial x_i} + (2/3) \frac{\partial q_i}{\partial x_i} + (2/3)P_{ij} \frac{\partial u_i}{\partial x_j} = 0 , \tag{1.4.3}$$

$$\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial}{x_k} (u_k \tau_{ij}) - (2/5)\frac{\partial q_i}{\partial x_j} + \tau_{ik}\frac{\partial u_j}{\partial x_k} - p\frac{\partial u_i}{\partial x_j} = - p \mu^{-1} \tau_{ij} , \tag{1.4.4}$$

$$\frac{\partial q_i}{\partial t} + \frac{\partial (u_k q_i)}{\partial x_k} + (7/5) q_k \frac{\partial u_i}{\partial x_k} + (2/5) q_k \frac{\partial u_k}{\partial x_i} + (2/5) q_i \frac{\partial u_k}{\partial x_k} - RT \frac{\partial \tau_{ik}}{\partial x_k}$$

$$- (7/2) \tau_{ik} R \frac{\partial T}{\partial x_k} + \tau_{ij} \rho^{-1} \frac{\partial P_{ik}}{\partial x_k} + (5/2) p R \frac{\partial T}{\partial x_i} = -(2/3)p \mu^{-1} q_i , \tag{1.4.5.}$$

where

$$\rho = m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f \, d v_{m1} \, dv_{m2} \, dv_{m3} \equiv m \int f \, d \vec{v}_m \equiv m n , \tag{1.4.6.}$$

(1.4.7.) $$f = f_o\left[1 - (2\,pRT)^{-1}\tau_{ij} V_i V_j - (pRT)^{-1} q_i V_i (1 - V^2 (5RT)^{-1})\right],$$

(1.4.8) $$f_o = (2\pi RT)^{-3/2}\, n \exp\left[- V^2 (2RT)^{-1}\right],$$

(1.4.9) $$u_i = n^{-1}\int v_{mi}\, f d\vec{v}_m, \qquad q_i = 2^{-1} m \int V_i V^2 f\, d\vec{v}_m,$$

(1.4.10) $$P_{ij} = m\int (v_{m_i} - v_{m_j})(v_{m_j} - v_{m_i})\, f d\vec{v}_m \equiv m\int V_i V_j f d\vec{v}_m \equiv p\delta_{ij} - \tau_{ij},$$

(1.4.11) $$\tau_{ij}^{(n)} = \mu \overline{\frac{\partial u_i}{\partial x_j}} - \mu p^{-1}\left[\frac{D\tau_{ij}^{(n-1)}}{Dt} + \tau_{ij}^{(n-1)}\frac{\partial u_k}{\partial x_k} - (2/5)\frac{\partial q_i^{(n-1)}}{\partial x_j} + \tau_{ik}^{(n-1)}\frac{\partial u_j}{\partial x_k}\right],$$

$$q_i^{(n)} = - k_{th}\frac{\partial T}{\partial x_i} - 3\mu(2p)^{-1}\Big(\frac{Dq_i^{(n-1)}}{Dt} + (7/5)\, q_i^{(n-1)}\frac{\partial u_j}{\partial x_j} - RT\frac{\partial \tau_{ij}^{(n-1)}}{\partial x_j}$$

$$+ q_j^{(n-1)}\left[\frac{\partial u_i}{\partial x_j} + (2/5)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]$$

(1.4.12) $$- (7/2)\tau_{ij}^{(n-1)} RT\frac{\partial T}{\partial x_j} + \tau_{ij}^{(n-1)}\rho^{-1}\frac{\partial P_{jk}^{(n-1)}}{\partial x_k}\Big),$$

and $\overline{A}_{ij} \equiv A_{ij} + A_{ji} - (2/3)\delta_{ij} A_{kk}$,

$$\mu = 0.243\,(2mA^{-1})^{1/2}\, mRT,$$

$$k_{th} = 15R\mu/4.$$

Ikenberry and Truesdell [37] present the n-th iterate:

$$(1.4.13) \quad -(p/\mu)P_{ij}^{(n+1)} \equiv 2pE_{ij} + \dot{P}_{ij}^{(n)}E - (2/3)P_{kl}^{(n)}E_{kl}\delta_{ij}$$

$$+ 2P_k^{(n)}({}_i u_j)k + \left[p_{ijk}^{(n)} - (2/3)h_k^{(n)}\delta_{ij}\right]_{,k},$$

$$-(p/2\mu)(3p_{ijk}^{(n+1)} - 2h_{(i\delta_{jk})}^{(n+1)}) \equiv \dot{p}_{ijk}^{(n)} + p_{ijk}^{(n)}E + p_{ijkl,1}^{(n)} -$$

$$-3p\rho^{-1}p_{,(i\delta_{jk})} - 3\rho^{-1}p_{,(i}P_{jk)}^{(n)}$$

$$(1.4.14) \qquad -3\rho^{-1}p\,\delta_{(ij}P_{k)1,1}^{(n)} - 3\rho^{-1}P_{(ij}^{(n)}P_{k)1,1}^{(n)} +$$

$$+3p_{1(iju_k),1}^{(n)},$$

where :

$$n \equiv \int_\infty f\,d\vec{\xi}, \quad \rho \equiv mn, \quad u_i \equiv \bar{\xi}_i, \quad c_i \equiv \xi_i - u_i;$$

$$p_{i_1 i_2 \ldots i_n} \equiv \rho\, \overline{c_{i_1} c_{i_2} \cdots c_{i_n}} = m\int_\infty c_{i_1} c_{i_2} \cdots c_{i_n} f\,d\vec{\xi}, \quad h_i = (1/2)\,p_{ijj}, \quad (1.4.15)$$

$$(1.4.16) \qquad (\dot{\ }) \equiv \frac{D(\ )}{Dt} \equiv \frac{\partial}{\partial t} + (\ )_{,i}u_i,$$

$$(1.4.17) \quad P_{ij} \equiv p_{ij} - p\delta_{ij}, \quad E \equiv u_{k,k}, \quad E_{ij} \equiv (1/2)(u_{i,j} + u_{j,i}) - (1/3)E\delta_{ij},$$

$$(1.4.18) \qquad \mu = (1/3)\,(2mG^{-1})^{1/2}\,mp\,(\rho A_2)^{-1},$$

G being the constant of proportionality between the intermolecular force and the reciprocal 5th power of distance, $A_2 = 1.37\ldots$ is a numerical constant evaluated by Maxwell [64] . The above iterate gives the following system of equations:

$$(1.4.19)\qquad \partial_t \rho^{(n+1)} + (\rho^{(n+1)} u_i^{(n+1)})_{,i} = 0 ,$$

$$(1.4.20)\qquad \rho^{(n)} \partial_t u_i^{(n+1)} + \rho^{(n)} u_{i,j}^{(n+1)} u_j^{(n)} + p_{,i}^{(n)} - \rho^{(n)} f_i = 0 ,$$

$$(1.4.21)\qquad 3\partial_t p^{(n+1)} + 3p_{,i}^{(n+1)} u_i^{(n)} + 5p^{(n+1)} E^{(n)} + 2P_{ij}^{(n+1)} E_{ij}^{(n)} + 2h_{i,i}^{(n)} = 0,$$

$$\partial_t P_{ij}^{(n+1)} + P_{ij,k}^{(n+1)} u_k^{(n)} + P_{ij}^{(n+1)} E^{(n)} + 2p^{(n)} E_{ij}^{(n)}$$

$$+ (p^{(n)}/\mu^{(n)}) P_{ij}^{(n+1)} - (2/3) P_{kl}^{(n+1)} E_{kl}^{(n)} \delta_{ij} + 2P^{(n+1)}_{k(i} w_{j),k}^{(n)}$$

$$\text{etc.}\,(1.4.22)\quad + P_{ijk,k}^{(n)} - (4/15)\, h_{k,k}^{(n)} \delta_{ij} + (4/5)\, h_{(i,j)}^{(n)} = 0 ,$$

The Hilbert-Enskog-Chapman-Burnett method obtains the necessary relations in terms of a solution of the Maxwell-Boltzmann equation. It is assumed that the collision term is dominant and that the distribution function may be determined by successive iterations on the collision term. This is roughly equivalent to assuming that the distribution function is given in terms of a power series in the mean free path. Then the Maxwell-Boltzmann equation can be reduced to a sequence of linear integral equations, which are in turn solved by replacing them by systems of simultaneous linear equations. The first approximation to the distribution function is the equilibrium distribution, (1.4.8) which gives the Euler equation after substitution into the Maxwell-Boltzmann equation.

The second and the third approximations respectively give the Navier-Stokes, and the Burnett equations.

The Euler, Navier-Stokes and Burnett equations are all contained in the system of Eqs. (1.4.1) - (1.4.5) if it is assumed that τ_{ij}, and q_i can be given in the form of (1.4.11) and (1.4.12).
$p_{ij}^{(o)} = q_i^{(o)} = 0$ (Euler equation) yields $\tau_{ij}^{(1)}$, $q_i^{(1)}$, which give the Navier-Stokes equations. $\tau_{ij}^{(2)}$, $q_i^{(2)}$ yield the Burnett equations (for Maxwellian molecules) and $\tau_{ij}^{(3)}$, $q_i^{(3)}$ will furnish the so-called Thirteen Moment equations, derived originally in 1949 completely independently by Grad [34].

1.5. Asymptotic Expansion.

Asymptotic solutions of Boltzmann equation are very thoroughly treated by Grad. The reader interested in the subject is referred to his works, (see [35, 36]).

1,6, MHD And Plasma

Actually both magneto-hydrodynamics and plasmadynamics in all their forms are used very extensively in the field of hypersonics. Below in chapter 2 and 4, we shall be barely able to consider a MHD flow. Plasmadynamics will not be discussed at all and the reader is referred to the enormous number of references on this subject. Needless to say, the dynamics of rarefied, ionized gas is today perhaps the most important aspect of the reentry hypersonics. Lack of space does not allow the author to discuss it as thoroughly as it should be done.

2. Hodograph Transformation and Integral Operators.

2.1. Preliminary Remarks.

During reentry of a blunt body there appears in front a bow -shock. Behind it the flow varies from sub- through trans-, to super-sonic one. The subsonic domain must be determined very precisely. It furnishes the initial data for the transonic flow and finally through it one must obtain precise information about the supersonic region at Mach number greater than one so that one can apply the theory of characteristics which is so powerful tool in the supersonic domain. We shall not discuss the corelation between subsonic and transonic flows.

In this chapter we restrict ourselves to discussing one of the possible techniques in the subsonic domain . Due to the high temperature the gas behind the bow-shock may be ionized. The technique in question is the hodograph transformation combined with the integral operator technique. The method is applicable at the present status to an inviscid, non-heat conducting gas. In the case of an irrotational steady flow, the equations in the hodograph plane are linear. In other cases of flows like rotational, magneto-hydrodynamic flow, etc., the equations in hodograph plane are nonlinear. In some terms there appears the Jacobian of the transformation in coefficients in front of derivatives. In such cases, one can apply a limiting process (iteration, successive approximation, etc.) starting with the irrotational equations. In hodograph plane such a procedure is much more efficient and more strongly convergent than in the physical plane. The procedure carries with itself all the disadvantages of the hodograph transformation technique like lack of