

# Quantum Computing by Practice

Python Programming in the Cloud  
with Qiskit and IBM-Q

—  
*Second Edition*  
—

Vladimir Silva

Apress®

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CARY, NC, USA

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*To my dear parents, Manuel and Anissia, and beloved siblings,  
Natasha, Alfredo, Sonia, and Ivan.*

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# About the Author

**Vladimir Silva** was born in Quito, Ecuador. He received a System's Analyst degree from the Polytechnic Institute of the Army in 1994. In the same year, he came to the United States as an exchange student pursuing an M.S. degree in Computer Science at Middle Tennessee State University. After graduation, he joined IBM as a software engineer. His interests include Quantum Computing, Neural Nets, and Artificial Intelligence. He also holds numerous IT certifications including OCP, MCSD, and MCP. He has written many technical books in the fields of distributed computing and security. His previous books include *Grid Computing for Developers* (Charles River Media), *Practical Eclipse Rich Client Platform Projects* (Apress), *Pro Android Games* (Apress), and *Advanced Android 4 Games* (Apress).

# About the Technical Reviewer



**Jason Whitehorn** is an experienced entrepreneur and software developer and has helped many companies automate and enhance their business solutions through data synchronization, SaaS architecture, and machine learning. Jason obtained his Bachelor of Science in Computer Science from Arkansas State University, but he traces his passion for development back many years before then, having first taught himself to program BASIC on his family's computer while still in middle school.

When he's not mentoring and helping his team at work, writing, or pursuing one of his many side-projects, Jason enjoys spending time with his wife and four children and living in the Tulsa, Oklahoma, region. More information about Jason can be found on his website: <https://jason.whitehorn.us>.

# Introduction

## The Quantum Computing Revolution

I wrote this book to be the ultimate guide for programming a quantum computer in the cloud. IBM has made their quantum rig (known as the IBM Quantum) available not only for research but for individuals, in general, interested in this exciting new field of computing.

Quantum computing is gaining traction and now is the time to learn to program these machines. In years to come, the first commercial quantum computers should be available, and they promise significant computational speedups compared to classical computers. Nowhere is this more apparent than in the field of cryptography where the quantum integer factorization algorithm can outperform the best classical solution by orders of magnitude, so much so that a practical implementation of this algorithm will render current asymmetric encryption useless.

All in all, this book is a journey of understanding. You may find some of the concepts explained throughout the chapters difficult to grasp; however, you are not alone. The great physicist Richard Feynman once said: *“If somebody tells you he understands quantum mechanics, it means he doesn’t understand quantum mechanics.”* Even the titans of this bizarre theory have struggled to comprehend what it all means.

I have tried to explore quantum computation to the best of my abilities by using real-world algorithms, circuits, code, and graphical results. Some of the algorithms included in this manuscript defy logic and seem more like voodoo magic than a computational description of a physical system. This is the main reason I decided to tackle this subject. Even though I find the mind-bending principles of quantum mechanics bizarre, I’ve always been fascinated by them. Thus, when IBM came up with its one-of-a-kind quantum computing platform for the cloud and opened it up for the rest of us, I jumped to the opportunity of learning and creating this manuscript.

Ultimately, this is my take on the subject, and I hope you find as much enjoyment in reading it as I did writing it. My humble advice: Learn to program quantum computers; soon they will be ever present in the data center, doing everything from search and simulations to medicine and artificial intelligence. Here is an overview of the manuscript’s contents.

## Chapter 1: Quantum Fields: The Building Blocks of Reality

It all began in the 1930s with Max Planck's reluctant genius. He came up with a new interpretation for the energy distribution of the light spectrum. He started it all by unwillingly postulating that the energy of the photon was not described by a continuous function, as believed by classical physicists, but by tiny chunks, which he called *quanta*. He was about to start the greatest revolution in science in this century: *quantum mechanics*. This chapter is an appetizer to the main course and explores the clash of two titans of physics: Albert Einstein and Niels Bohr. Quantum mechanics was a revolutionary theory in the 1930s, and most of the scientific establishment was reluctant to accept it, including the colossus of the century: Albert Einstein. Fresh from winning the Nobel Prize, Einstein never accepted the probabilistic nature of quantum mechanics. This caused a rift with its biggest champion: Niels Bohr. The two greats debated it out for decades and never resolved their differences. Ultimately, quantum mechanics has withstood 70 years of theoretical and experimental challenges, to emerge always triumphant. Read this chapter and explore the theory, experiments, and results, all under the cover of the incredible story of these two extraordinary individuals.

## Chapter 2: Richard Feynman, Demigod of Physics, Father of the Quantum Computer

In the 1980s, the great physicist Richard Feynman proposed a quantum computer. That is a computer that can take advantage of the principles of quantum mechanics to solve problems faster. The race is on to construct such a machine. This chapter explores, in general terms, the basic architecture of a quantum computer: qubits – the basic blocks of quantum computation. They may not seem like much but they have almost magical properties: Superposition, believe it or not, a qubit can be in two states at the same time: 0 and 1. This is a concept hard to grasp at the macroscale where we live. Nevertheless, at the atomic scale, all bets are off. This fact has been proven experimentally for over 70 years. Thus, superposition allows a quantum computer to outmuscle a classical computer by performing large amounts of computation with relatively small numbers of qubits. Another mind-bender is qubit entanglement: entangled qubits transfer states, when observed, faster than the speed of light across time or space! Wrap your head

around that. All in all, this chapter explores all the physical components of a quantum computer: quantum gates, types of qubits such as superconducting loops, ion traps, topological braids, and more. Furthermore, the current efforts of all major technology players in the subject are described, as well as other types of quantum computation such as quantum annealing.

## **Chapter 3: Behold, the Qubit Revolution**

In this chapter, we look at the basic architecture of the qubit as designed by the pioneering IT companies in the field. You will also learn that although qubits are mostly experimental and difficult to build, it doesn't mean that one can't be constructed with some optical tools and some ingenuity. Even if a little crude and primitive, a quantum gate can be built using refraction crystals, photon emitters, and a simple budget. This chapter also explores superconducting loops as the de facto method for building qubits along with other popular designs and their relationship to each other.

## **Chapter 4: Enter IBM Quantum: A One-of-a-Kind Platform for Quantum Computing in the Cloud**

In this chapter, you will get your feet wet with the IBM Q Experience. This is the first quantum computing platform in the cloud that provides real or simulated quantum devices for the rest of us. Traditionally, a real quantum device will be available only for research purposes. Not anymore, thanks to the folks at IBM who have been building this stuff for decades and graciously decided to open it up for public use.

Learn how to create a quantum circuit using the visual composer or write it down using the excellent Python SDK for the programmer within you. Then execute your circuit in the real thing, explore the results, and take the first step in your new career as a quantum programmer. IBM may have created the first quantum computing platform in the cloud, but its competitors are close behind. Expect to see new cloud platforms soon from other IT giants. Now is the time to learn.



## **Chapter 5: Mathematical Foundation: Time to Dust Up That Linear Algebra**

Matrices, complex numbers, and tensor products are the holy trinity of quantum computing. The bizarre properties of quantum mechanics are entirely described by matrices. It is the rich interpretation of matrices and complex numbers that allows for a bigger landscape resulting in an advantage over traditional scalar-based mathematics. Quantum mechanics sounds and looks weird but at the end is just fancy linear algebra.

## **Chapter 6: Qiskit, Awesome SDK for Quantum Programming in Python**

Qiskit stands for Quantum Information Software Kit. It is a Python SDK to write quantum programs in the cloud or a local simulator. In this chapter, you will learn how to set up the Python SDK on your PC. Next, you will learn how quantum gates are described using linear algebra to gain a deeper understanding of what goes on behind the scenes. This is the appetizer to your first quantum program, a very simple thing to familiarize you with the syntax of the Python SDK. Finally, you will run it in a real quantum device. Of course, quantum programs can also be created visually in the composer. Gain a deeper understanding of quantum gates, the basic building blocks of a quantum program. All this and more is covered in this chapter.

## **Chapter 7: Start Your Engines: From Quantum Random Numbers to Teleportation and Super Dense Coding**

This chapter is a journey through three remarkable information-processing capabilities of quantum systems. Quantum random number generation explores the nature of quantum mechanics as a source of true randomness. You will learn how this can be achieved using very simple logic gates and the Python SDK. Next, this chapter explores two related information processing protocols: super dense coding and quantum teleportation. They have exuberant names and almost magical properties. Discover their secrets, write circuits for the composer, execute remotely using Python, and finally interpret and verify their results.

## **Chapter 8: Game Theory: With Quantum Mechanics, Odds Are Always in Your Favor**

Here is a weird one: this chapter explores two game puzzles that show the remarkable power of quantum algorithms over their classical counterparts – the counterfeit coin puzzle and the Mermin-Peres Magic Square. In the counterfeit coin puzzle, a quantum algorithm is used to reach a quartic speed up over the classical solution for finding a fake coin using a balance scale a limited number of times. The Mermin-Peres Magic Square is an example of quantum pseudo-telepathy or the ability of players to almost read each other's minds, achieving outcomes only possible if they communicate during the game.

## **Chapter 9: Quantum Advantage with Deutsch-Jozsa, Bernstein-Vazirani, and Simon's Algorithms**

This chapter looks at three algorithms of little practical use but important, because they were the first to show that quantum computers can solve problems significantly faster than classical ones: Deutsch-Jozsa, Bernstein-Vazirani, and Simon's algorithms. They achieve significant performance boost via massive parallelism by using the Hadamard gate to put the input in superposition. They also illustrate critical concepts such as oracles or black boxes that perform some transformation on the input, and phase kickback, a powerful technique used to alter the phase of inputs so they can cancel each other.

## **Chapter 10: Advanced Algorithms: Unstructured Search and Integer Factorization with Grover and Shor**

This chapter showcases two algorithms that have generated excitement about the possibilities of practical quantum computation: Grover's Search, an unstructured quantum search algorithm capable of finding inputs at an average of the square root of  $N$  steps. This is much faster than the best classical solution at  $N/2$  steps. It may not seem that much, but when talking about very large databases, this algorithm can crush it in the data center. Expect all web searches to be performed by Grover's in the future. Shor's

## INTRODUCTION

Integer Factorization: the notorious quantum factorization that experts say could bring current asymmetric cryptography to its knees. This is the best example of the power of quantum computation by providing exponential speedups over the best classical solution.

# Chapter 11: Quantum in the Real World: Advanced Chemistry and Protein Folding

Quantum is already working hard to make a difference in the fields of Chemistry and Medicine. This chapter showcases two amazing real-life experiments that illustrate its power: ground states are important in molecular chemistry, with most elements modeled using lattices where vertices represent interacting atoms. In this chapter, you will learn how to minimize the energy Hamiltonian of a molecule to reach its ground state using lattices. Next, proteins are the fundamental building blocks that power all life. Reliably predicting protein structures is extremely complicated and can change our understanding of nature. In this experiment, you will learn about protein amino acids, peptides, chains, nomenclature, and more; and best of all, you will learn how its structure can be predicted using a quantum computer.

## CHAPTER 1

# Quantum Fields: The Building Blocks of Reality

The beginning of the 20th century, more specifically 1930s Europe, witnessed the dawn of arguably one of the greatest theories in human history: quantum mechanics. After almost a century of change, this wonder of imagination has morphed and taken many directions. One of these is *quantum field theory* (QFT) which is the subject of this chapter. If you enjoy physics and wish to understand why things are the way they are, then you must get your feet wet with QFT. It has been called the most successful theory in history, riding high since the 1950s and giving rise to the standard model of particle physics. This is the modern view of how nature works at the smallest scale, being proven right time and again by countless experiments and instruments like the Large Hadron Collider (LHC). All in all, the story of how QFT came to be, and the Masters of Physics behind it, is a tale of wonder, furious rivalry but ultimate collaboration.

Our story begins in 1900 when Lord Kelvin stood in front of the British Science Royal Society and enunciated: “*There is nothing else to be discovered in physics*” – a powerful statement at the time but clearly wrong in hindsight. Perhaps, we should thank the lord for such a bold proclamation because it is statements like that that drive others to prove them wrong. This was put to the test 30 years later in Germany.

Around the 1930s, the great German physicist Max Planck (1858–1947) was working on the black-body radiation problem, more specifically in the ultraviolet catastrophe. To understand this problem, let’s backtrack to the physics of how materials glow in multiple colors at different temperatures. In 1900 British physicist Lord Rayleigh derived an approximation to predict that process. To accomplish his task, Rayleigh used the so-called black body, a simple object that would absorb and emit light but not reflect it.

Note that the term *black* doesn't mean its color is black but that it simply absorbs and emits light but *does not reflect it*, so when observed, you'll see its glow or *radiation*.

Rayleigh's work is known as the Rayleigh-Jeans law for spectral radiation of a black body as a function of its wavelength  $\lambda$  (lambda) and its temperature in Kelvin degrees (K) (see Equation 1.1):

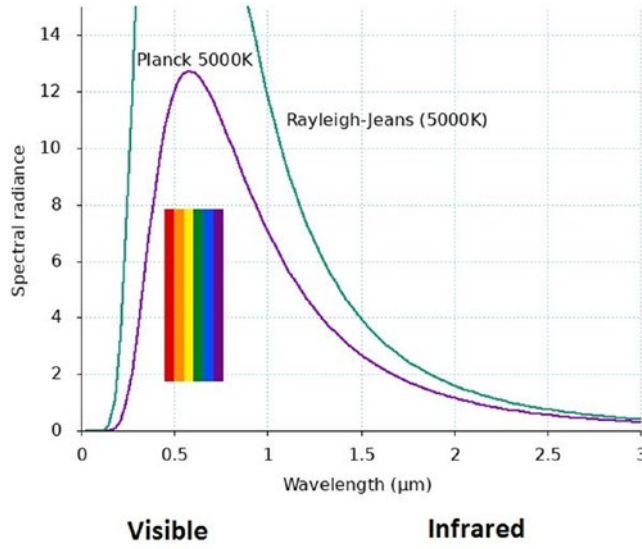
$$B_{\lambda}(T) = \frac{2cK_B T}{\lambda^4} \quad (1.1)$$

where

- $c$  = speed of light (299792458 m/s)
- $K_B$ , the Boltzmann constant =  $1.38064852 \times 10^{-23}$  m<sup>2</sup> kg s<sup>-2</sup> K<sup>-1</sup>
- $\lambda$  = wavelength
- $T$  = temperature in Kelvin degrees

## Enter Max Planck, the Father of Quantum Mechanics

The Rayleigh-Jeans law works great for higher wavelengths (in the infrared spectrum outside of visible light) but gives infinite values in the visible spectrum. Figure 1-1 shows a graph of the Rayleigh-Jeans spectral radiance for wavelengths of visible and infrared for a black body at 5000 degrees Kelvin. This is what is known as the ultraviolet catastrophe: the infinite values of radiation of light in the visible spectrum as predicted by classical physics. This is simply not possible; if this was true, then we'll all get cooked up by simply getting close to a candle light! Max Planck realized this and found a solution in the 1930s earning him a Nobel Prize and a place in history.



**Figure 1-1.** Graph of the Rayleigh-Jeans law vs. Planck's solution for the ultraviolet catastrophe

Planck altered Rayleigh's original derivation by changing the formula to match experimental results as shown in Equation 1.2.

$$B_{\lambda}(T) = \frac{2cK_B T}{\lambda^4} \quad (1.1)$$

$$B_{\lambda}(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda K_B T}} - 1} \quad (1.2)$$

where  $h$  is Planck's constant =  $6.62 \times 10^{-34}$  m<sup>2</sup>kg/s.

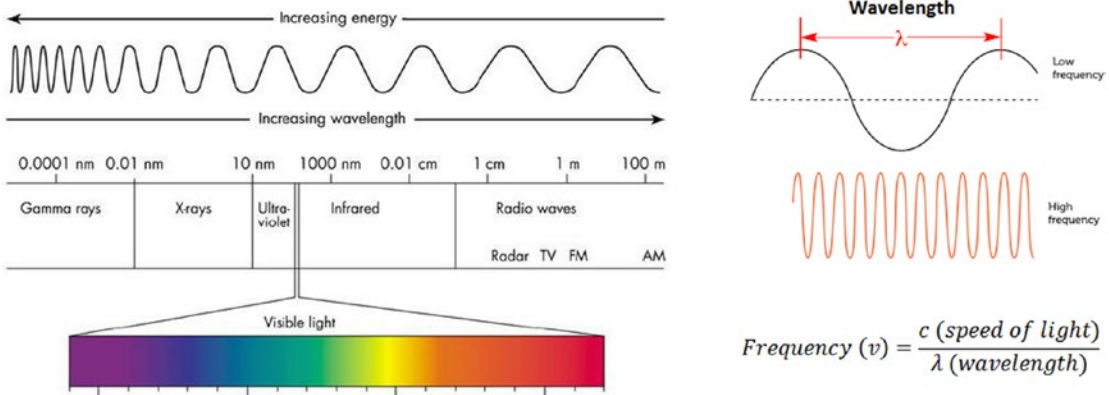
He made an incredible assumption for the time: energy can be emitted or absorbed in discrete chunks which he called *quanta*:  $E = h\nu = h \frac{c}{\lambda}$  where  $\nu$  is the frequency. Note that frequency equals the speed of light divided by the wavelength  $\nu = \frac{c}{\lambda}$ . This may seem trivial nowadays, but in the 1930s was ground-breaking; not even Planck fully understood what he had unleashed. He gave birth to a brand new theory: quantum mechanics.

# Planck Hits the Jackpot, Einstein Collects a Nobel Prize

So at the time, Planck didn't realize how huge his postulate of *energy quanta* was, as he admitted that his solution for the ultraviolet catastrophe was simply a workaround for the maths of the Rayleigh-Jeans law to make it fit well-known experimental results. To grasp the power of this postulate, one must look at the view of the nature of light pre-post Planck's era.

## The Nature of Light Before Planck

Since the 19th century, it was well accepted that light behaved like a wave. Scottish physicist James Clerk Maxwell (1831–1879) provided a description of the fundamental properties of such waves (see Figure 1-2).



**Figure 1-2.** The nature of light in the 19th century

- A fundamental property of a light wave is its wavelength or lambda ( $\lambda$ ).
- Look at the right side of Figure 1-2: At very short wavelengths, we have lots of waves; the reverse is also true at higher wavelengths. This is the frequency ( $\nu$ ), a second fundamental property of light waves.

It seems logical to assume that at high frequencies (short wavelengths), the energy of the wave is higher (as there is more stuff flowing in) and that at lower frequencies (higher wavelengths) the energy decreases. Therefore the energy ( $E$ ) is directly proportional to

its frequency ( $\nu$ ) and inversely proportional to its wavelength ( $\lambda$ ). This knowledge gave rise to the standard spectrum of light in the 19th century:

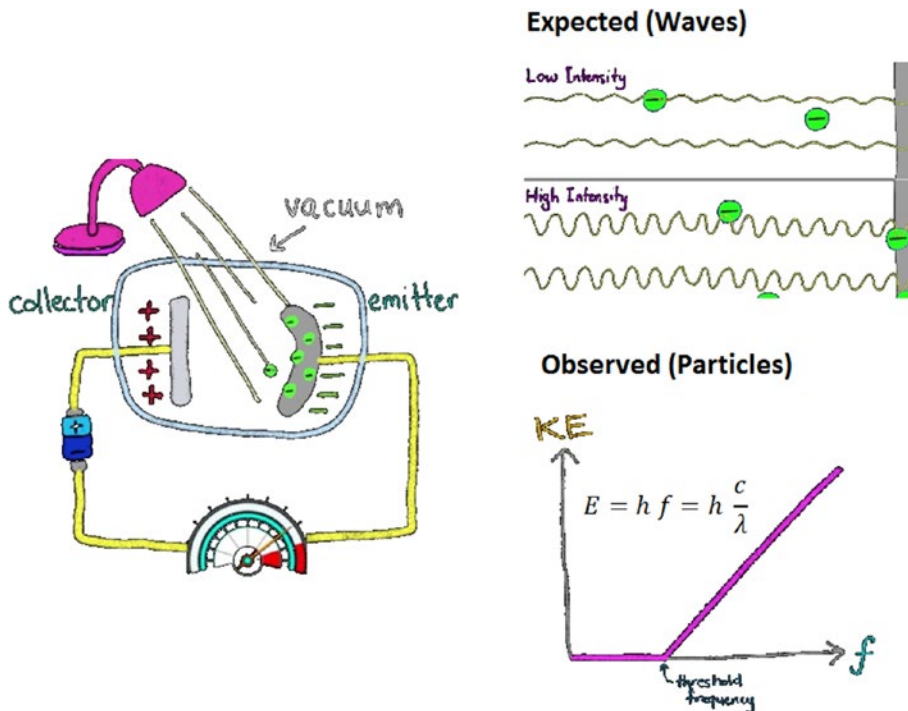
- On the left side of the spectrum (at the shortest wavelengths between 1 picometer and 0.01 nanometers [nm]), sit the *gamma rays*: very dangerous, the usual result of a supernova explosion, they are the most energetic. A gamma-ray burst from a supernova can destroy everything in its path: all life on Earth, for example, even the solar system. You don't want to be in the crosshairs of a gamma-ray burst!
- Next, at a wavelength of 0.01–10nm, sit the well-known x-rays: very helpful for looking inside of things: organic or inorganic, but still dangerous enough to cause cancer over persistent exposure.
- At a wavelength of 10–400 nm, we have ultraviolet light (UV): this is the radiation from the sun that gives life to our Earth but can be harmful in high doses. Lucky for us, the ozone layer on Earth keeps the levels in balance to make life possible.
- At a tiny sort after the UV range sits the visible light spectrum that allows us to enjoy everything we see in this beautiful universe.
- Next, infrared at wavelengths up to 1050 nanometers. It is used in industrial, scientific, military, law enforcement, and medical applications. In such devices as night vision goggles, heat sensors, and others.
- Finally, radio waves above the infrared range. These are used by most human technology to send all kinds of information such as audio, video, TV, radio, cell phones, you name it.

## After Planck, Physics Will Never Be the Same

In the 1930s Planck turned the classical understanding of the nature of light upside down. Even though his postulate of energy quanta was dubbed lunacy by most physicists and remained unnoticed for years, it will take another giant of the century, Albert Einstein, to seize on this discovery and come up with a brand new interpretation of light. Thus, the photon was born.



This is not well known to most people, but Einstein didn't win a Nobel Prize for his masterpiece on *The Theory of Relativity*, but for his work on the quantum nature of light and the photoelectric effect. Using Planck's idea, Einstein imagined light as discrete waves (particles) which he called photons. He used this to solve a paradox in the photoelectric effect unknown at the time (see Figure 1-3).



**Figure 1-3.** A fresh idea on the photoelectric effect earned Einstein the Nobel Prize in Physics in 1921

As its name indicates, the photoelectric effect seeks to describe the behavior of electrons over a metal surface when light is thrown in the mix. To this end, the experiment in Figure 1-3 was devised:

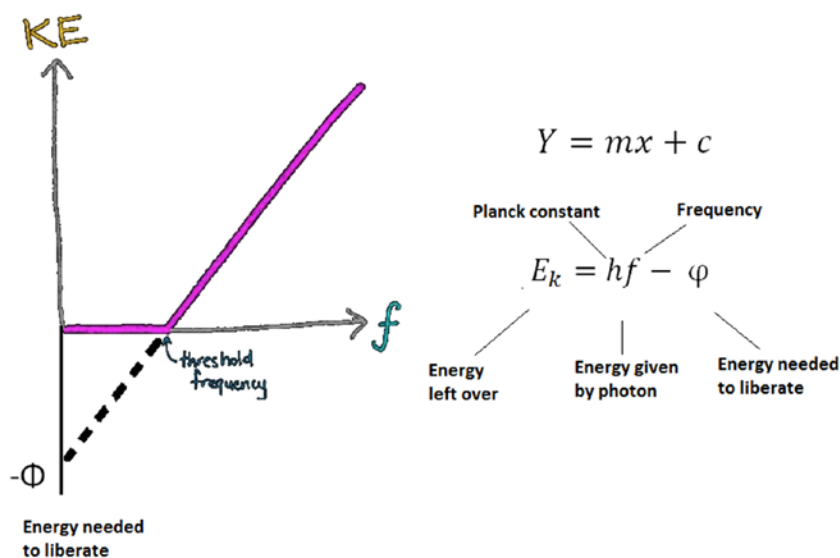
- Start with two metal plates. Let's call them the emitter and the collector. Both are attached via a cable to a battery. The negative end of the battery is connected to the emitter, and the positive to the collector.

- As we all know, electrons have a negative charge; thus, they flow to the emitter while the positive charge gathers in the collector. Remember also that opposite charges attract.
- The idea is to measure the kinetic energy of the electrons when they flow from the emitter to the collector when a light source is thrown into the emitter. To achieve this accurately, a vacuum is set among the two.
- If light flows as a wave as classical physics demands, then when the light hits the electrons, they will become energized and escape the surface of the emitter toward the collector. Furthermore, as the intensity (the amount) of light is increased, more electrons will get energized and escape in larger quantities. This increase in charge can be measured by the gauge as shown.

However, this is not what happens. Two things were observed in reality:

1. The increase in charge (the kinetic energy of the electrons) does not depend on the intensity of the light but on its frequency.
2. Even stranger, not all frequencies energize the electrons to escape the emitter. If we were to draw the kinetic energy (KE) as a function of the frequency ( $f$ ) (see the lower right side of Figure 1-3), then there is a point in the curve (threshold frequency) after which the electrons escape. Values below this threshold and the electrons remain unchanged. This is a puzzle indeed!

Einstein proposed a solution to this puzzle: by postulating that energy behaves as a particle, he solved the paradox of item 2 of the list. Imagine that you are at the county fair looking to win a prize by knocking down pins with a ball. If you throw marbles at the pins, they won't budge; however, throw a baseball, and the pins will be knocked down earning you that desired prize. This is what Einstein thought occurred in this situation. Low frequency photons don't have enough energy to power up the electrons to escape the emitter. Increase the frequency of the light; it increases the energy of the electrons so they escape generating a current that can be measured. From this, a mathematical model can be derived (see Figure 1-4).



**Figure 1-4.** Equation for the photoelectric effect

Figure 1-4 shows a graph of the kinetic energy of the electron (EK) as a function of the light frequency (f). At low frequencies, no electrons escape until the threshold frequency is reached. Now, extend the line as shown by the dotted track in the figure, and we have a straight line graph (note that the point at which the dotted track intersects the Y axis is named by the Greek letter  $\phi$  (Phi)). This is the energy needed to liberate the electron. Thus, this line graph can be described by the algebraic equation  $Y = mx + c$  where  $m$  is the gradient and  $c$  is the Y-intercept.

Now instead of Y, substitute the kinetic energy, with the gradient  $m$  being Planck's constant ( $h$ ), the frequency (f) instead of x, and  $c$  being the energy needed to liberate or  $-\phi$ . Therefore, our line graph equality becomes  $E_k = hf - \phi$ .

This is the equation for the photoelectric effect: the energy leftover after the electron is liberated equals the energy given by the photon minus the energy needed to liberate it.

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**Tip** Incidentally, the first scientist to think of light as a particle was Isaac Newton. He thought light traveled in small packets which he called co-puzzles. He also thought these packets had mass; something that is incorrect. Unfortunately, this idea never took off and lay dormant until it was revived by the Planck-Einstein revolution of the 1930s.

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# Quantum Mechanics Comes in Many Flavors

There is little doubt that the 1930s were the golden age of physics in the 20th century. Nobel prizes were awarded like candy, and it seemed that nothing could stop humanity in its quest to unravel the secrets of nature. Since then, quantum mechanics has stood tall for almost a century of endless theoretical and experimental challenges. All in all, it has seen a good deal of change over the years. These are the so-called interpretations of quantum mechanics, and they come in really bizarre flavors.

## Copenhagen Interpretation

This is the earliest consensus about the meaning of quantum mechanics, and was born out of the golden age of physics with contributions from Max Planck, Niels Bohr, Werner Heisenberg, and others in Copenhagen during the 1920s.

## The Revolution Begins with Planck, Bohr, and Schrödinger

Max Planck's postulate of energy quanta started the revolution with contributions by Einstein on the duality and/or quantum nature of light. That is, the idea that light behaves as both a wave and a particle.

Danish physicist Niels Bohr (1885–1962) funded the Institute of Theoretical Physics in Copenhagen in the 1920s to work on the brand-new field of atomic research. At the time, the atom was thought to look like a tiny solar system with a nucleus at the center made of protons, neutrons, and electrons orbiting around. This was known as the Rutherford model, but it had a terrible problem: electric charge! If the negatively charged electrons orbit around the positively charged nucleus, then as opposite charges attract, the electrons will eventually collapse into the nucleus destroying all matter in existence. Bohr foresaw this situation and used Planck's idea of energy quanta to theorize that electrons jump from one orbit to another by gaining or losing energy; something that he called a *quantum jump*. This idea later became known as the Bohr atom, but it had a weird characteristic: electrons didn't simply travel from one orbit to another. They instantaneously disappear from one orbit and reappear in another. This did not sit well with another colossus of physics: Erwin Schrödinger.

Austrian physicist Erwin Schrödinger (1887–1961) is the father of the famous wave function  $\psi$  (Cyrillic - Psi). Schrödinger was looking to describe the energy of a physical system; he came up with what is now considered the most powerful tool in physics in the last century (see Figure 1-5).

The diagram shows the Schrödinger wave equation: 
$$\frac{\delta^2 \psi}{\delta x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$
 Labels with arrows point to various parts of the equation:
 

- 'Second derivative' points to  $\delta^2 \psi$ .
- 'Probability' points to the vertical axis of the graph below.
- 'Position' points to  $x$  in  $\delta x^2$ .
- 'Mass' points to  $m$ .
- 'Potential Energy' points to  $V$ .
- 'Energy' points to  $E$ .
- 'Planck constant' points to  $h$ .

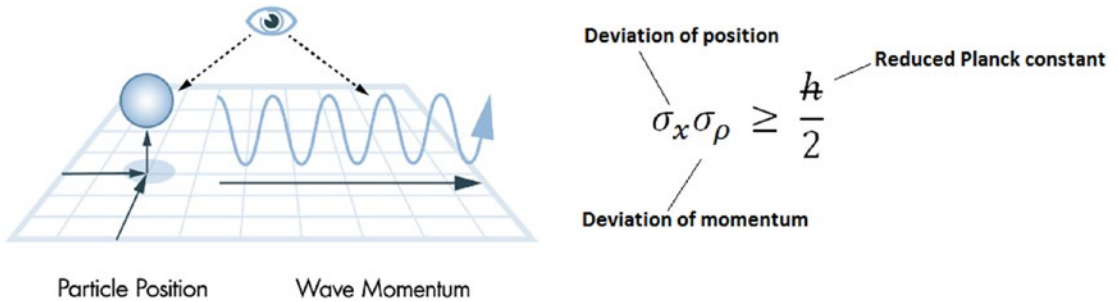
 Below the equation is a graph with a vertical axis labeled 'Probability' and a horizontal axis labeled 'Position'. A blue bell-shaped curve represents the wave function  $\psi$  centered on the position axis.

**Figure 1-5.** Schrödinger wave function  $\psi$  is the cornerstone of quantum mechanics

Schrödinger detested Bohr's interpretation of the atom famously stating that "If I am to accept the quantum jump, then I am sorry I ever got into the field of atomic research." As a matter of fact, his wave function was an attempt to defeat Planck-Bohr-Einstein. He wanted to throw away the nascent theory of energy quanta and return to the continuous classical model of wave physics, even pushing the idea that all reality can be described entirely by waves. So why is  $\psi$  used nowadays everywhere in quantum mechanics? Thank this to our next physicist: Max Born.

German-Jewish physicist Max Born (1882–1970) took Schrödinger's wave function in an entirely new direction. Born proposed a probabilistic interpretation of  $\psi$ , that is, the state of a particle exists in constant flux, and the only thing we can know is the probability of the particle at a given state. Born postulated that this probability is  $P = \psi^2$ . Needless to say, Schrödinger didn't like this at all as he thought his wave function was being misused. He took a swing at Born with his now famous thought experiment: the quantum cat. But before we check if the cat in the box is dead or alive and why, consider this witty story: In the quintessential American cartoon *Futurama* (by Matt Groening – creator of *The Simpsons*), our hero Fry enrolls in the police academy in New-NewYork on Earth in the year 3000. One day while on patrol, Fry chases a bandit carrying a mysterious box in the trunk of his car. Once in custody, the bandit is revealed to be Werner Heisenberg. Fry looks at the box with a face full of trepidation and asks: "What's in the box?" To which Heisenberg replies, "a cat." "Is the cat dead or alive?" asks Fry. Heisenberg replies: "the cat is neither dead nor alive but in a superposition of states with a probability assigned to each." Long story short, Heisenberg the bandit is arrested as a major violator of the laws of physics. This was a funny tale for the physics buff. Nevertheless, it shows the quantum cat has become folklore, and the prime example used to explain the probabilistic nature of quantum mechanics.

The powerful *Heisenberg's uncertainty principle (HUP)* is the work of German physicist Werner Heisenberg (1901–1976), and it is one of the foundations of quantum theory. It describes a degree of uncertainty in the relationship between the position ( $x$ ) and the momentum ( $p$ ) of a particle. More clearly, we can measure the exact *position* or *momentum* of a particle but not both. The uncertainty principle arises from the fundamental wave-matter duality of quantum objects (see Figure 1-6).



**Figure 1-6.** Uncertainty is a fundamental property of the wave-particle duality of a quantum object's complementary variables such as position and momentum

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**Tip** A remarkable point is that at the beginning, this degree of uncertainty was confused with the *observer effect*, which states that the act of measurement alters the state of a quantum system. As a matter of fact, Heisenberg himself used the observer effect as a physical explanation of this postulate. Since then this has been proven untrue with a rigorous mathematical derivation of HUP provided by physicist Earle Hesse Kennard in 1928.

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The uncertainty principle has a profound effect in the world of thermodynamics: for example, it gave rise to the notion of *zero-point energy*. In the Kelvin scale of temperature, zero kelvin is called the *absolute zero* or the temperature at which all molecular activity stops. This fact is forbidden by quantum mechanics and the uncertainty principle because, if all molecular activity ceases, then the position and momentum of a particle will be known. This is not possible; you either know the position or the momentum of a particle but not both. Thus even at absolute zero, particles are vibrating with a tiny amount of energy, hence the term zero-point energy.